

ELECTRIC MACHINES – 1

EE-352

TEXT BOOK

ELECTRIC MACHINERY SIXTH EDITION

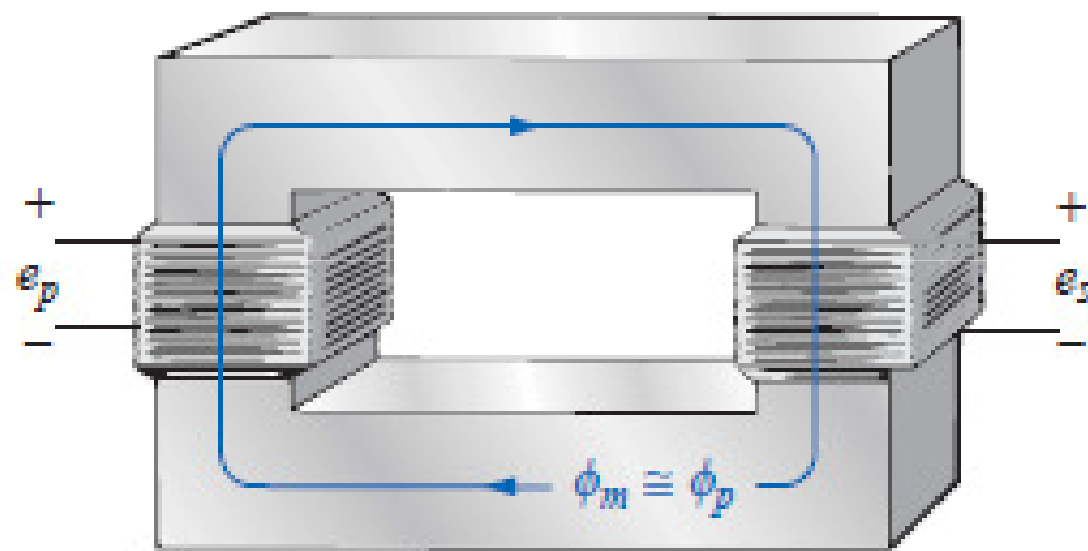
BY: Fitzgerald, kingsley, and UMANS

TRANSFORMERS

- *Transformers are used to:*
- *1- Change the voltage magnitude at the same frequency.*
- *2- Change current magnitudes, at the same frequency.*
- *3- Change apparent impedance, as used for impedance matching.*
- *4- Isolate dc current from ac current, as in electronic circuits.*

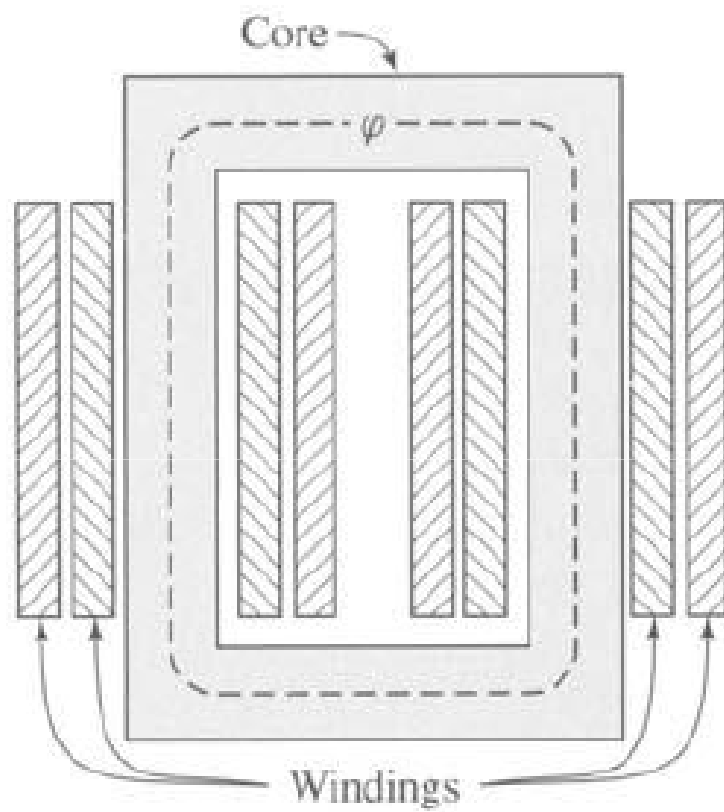
Construction

- *Two or more windings of suitable number of turns, made from either copper or aluminum conductors, isolated from each other.*
- *Iron core made of thin laminations of high magnetic quality steel in order to increase the mutual magnetic flux between transformer windings.*
- *Tank or case in which these components are placed.*

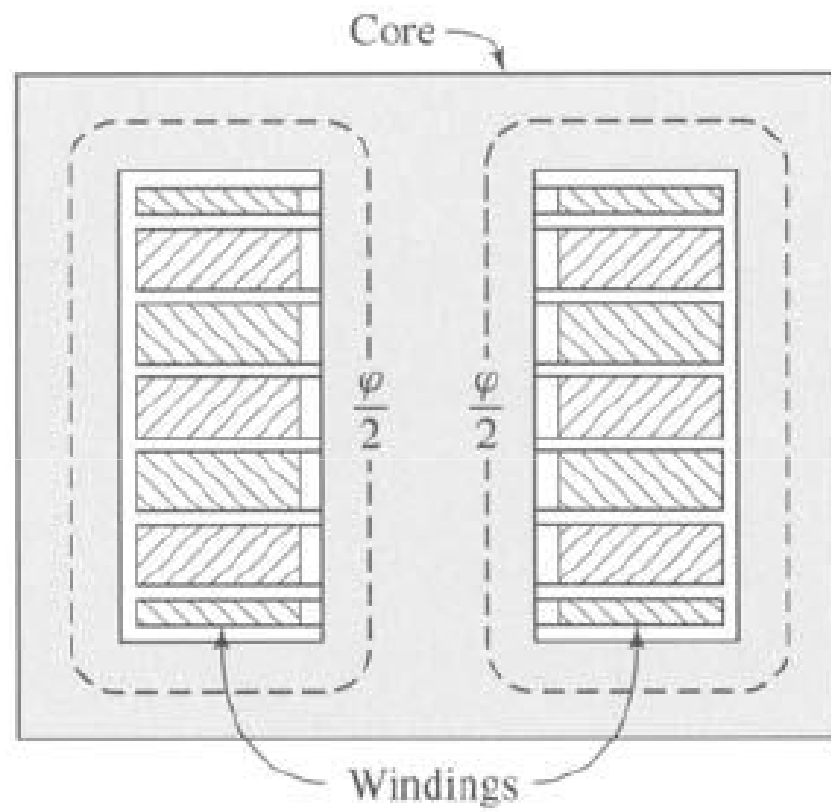


Steel core

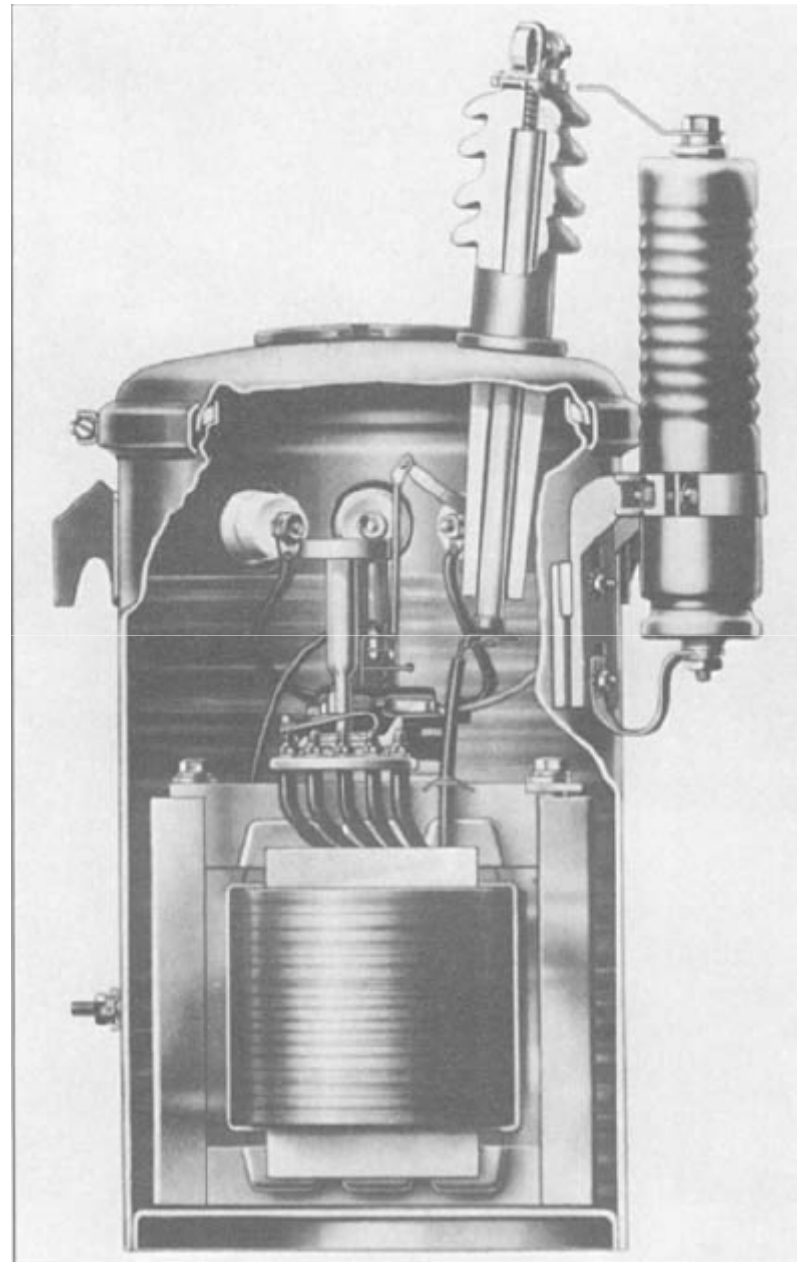
- *In order to increase the mutual coupling between transformer windings, and to decrease leakage flux, transformer windings are placed as close as possible to each other as shown.*



(a)



(b)

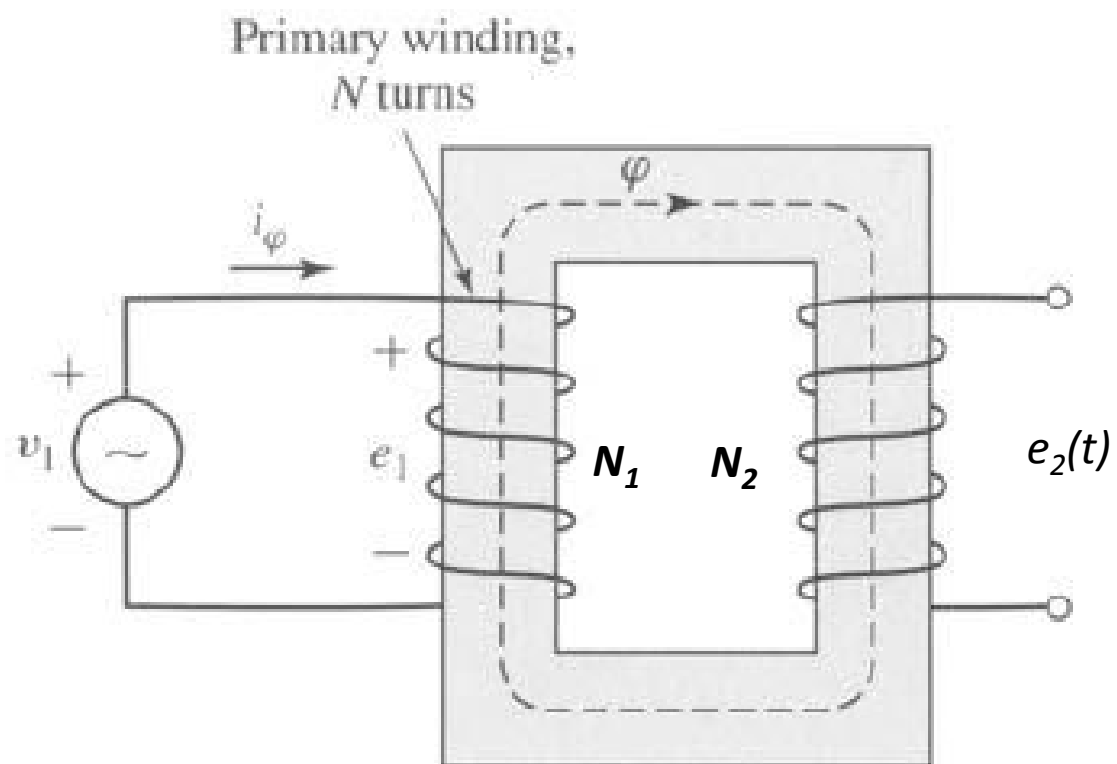


Ideal transformer

- *Iron core is made of magnetic material of infinite permeability. So its core reluctance is zero.*
- *No leakage flux. So all primary flux links all secondary winding.*
- *No losses. No core losses, and no copper losses.*

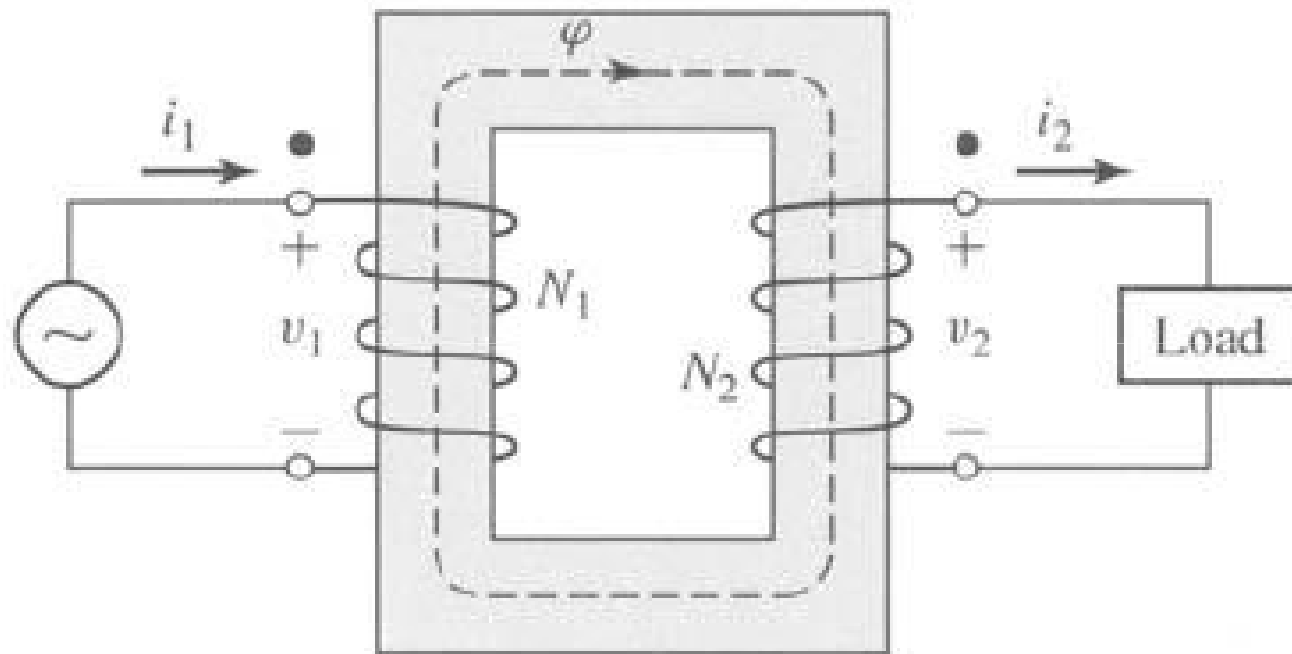
Voltage equations

- *Ac voltage connected to the primary, will establish a sinusoidal flux in transformer that links both windings. So*
- $\lambda_1 = N_1 \varphi$ and $\lambda_2 = N_2 \varphi$
- $e_1(t) = \frac{d}{dt} \lambda_1 = N_1 \frac{d}{dt} \varphi$ and $e_2(t) = N_2 \frac{d}{dt} \varphi$
- Then $\frac{e_1(t)}{e_2(t)} = \frac{N_1}{N_2}$
- *This shows that transformer changes voltage in direct proportion to the turns ratio.*



Current relations

- *Since iron core reluctance is zero, then*
- $\mathcal{F} = \mathcal{R} \cdot \varphi = N_1 \cdot i_1 - N_2 \cdot i_2 = 0$
- *Then* $\frac{i_1}{i_2} = \frac{N_2}{N_1}$
- *This shows that transformers change current in inverse proportion to turns ratio. That is to say that currents are changed in reverse of change of voltages.*



EXAMPLE For the iron-core transformer of Fig. 21.5:

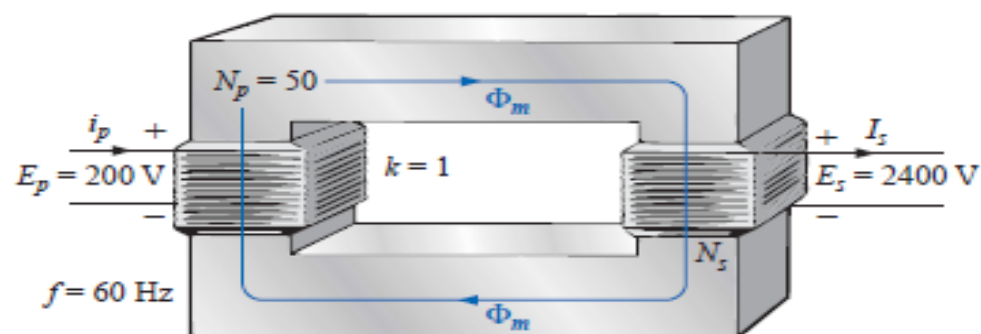


FIG. 21.5
Example 21.2.

- a. Find the maximum flux Φ_m .
- b. Find the secondary turns N_s .

Solutions:

a. $E_p = 4.44N_pf\Phi_m$

$$\text{Therefore, } \Phi_m = \frac{E_p}{4.44N_pf} = \frac{200 \text{ V}}{(4.44)(50 \text{ t})(60 \text{ Hz})}$$

and

$$\Phi_m = \mathbf{15.02 \text{ mWb}}$$

b. $\frac{E_p}{E_s} = \frac{N_p}{N_s}$

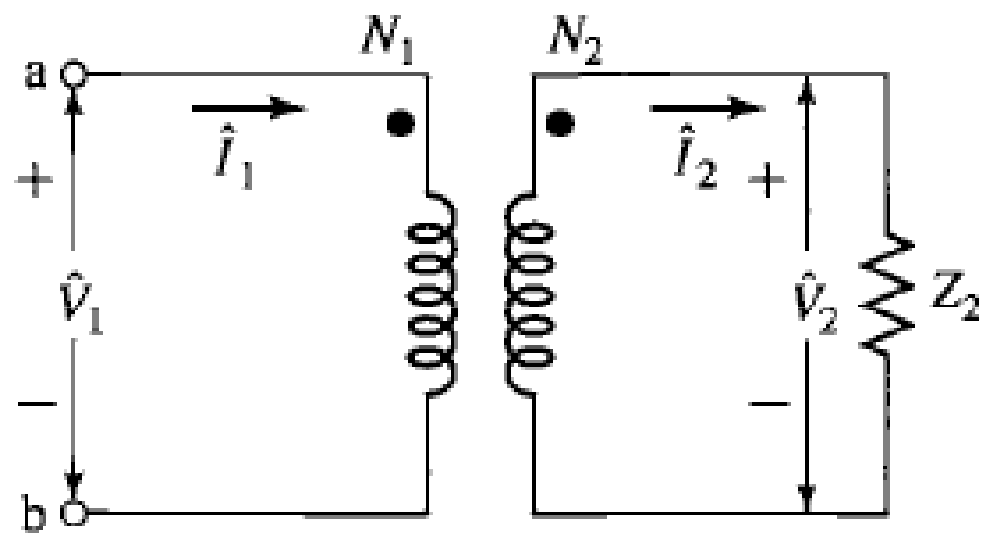
$$\begin{aligned} \text{Therefore, } N_s &= \frac{N_p E_s}{E_p} = \frac{(50 \text{ t})(2400 \text{ V})}{200 \text{ V}} \\ &= \mathbf{600 \text{ turns}} \end{aligned}$$

Impedance relations

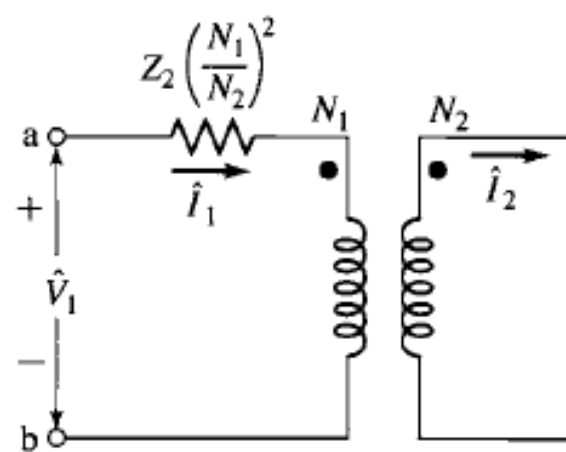
- *From previous relations, and from figure shown, the apparent impedance as seen at the primary winding is:*

- $$Z_1 = \frac{v_1}{i_1} = \frac{\left(\frac{N_1 \cdot v_2}{N_2}\right)}{\left(\frac{N_2 \cdot i_2}{N_1}\right)} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$$

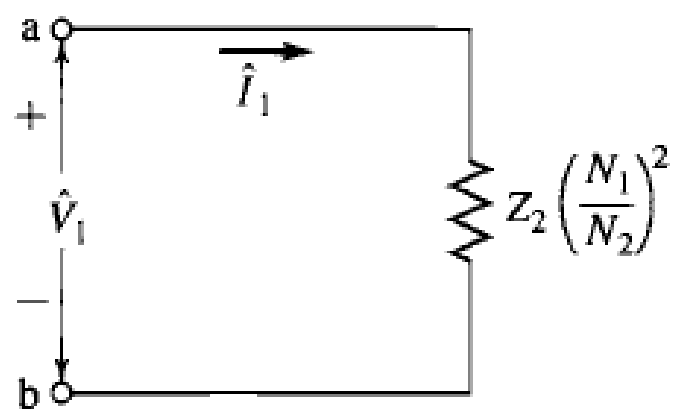
- *That shows that transformers change impedance in proportion to turns ratio squared.*



(a)



(b)



(c)

Transformer types

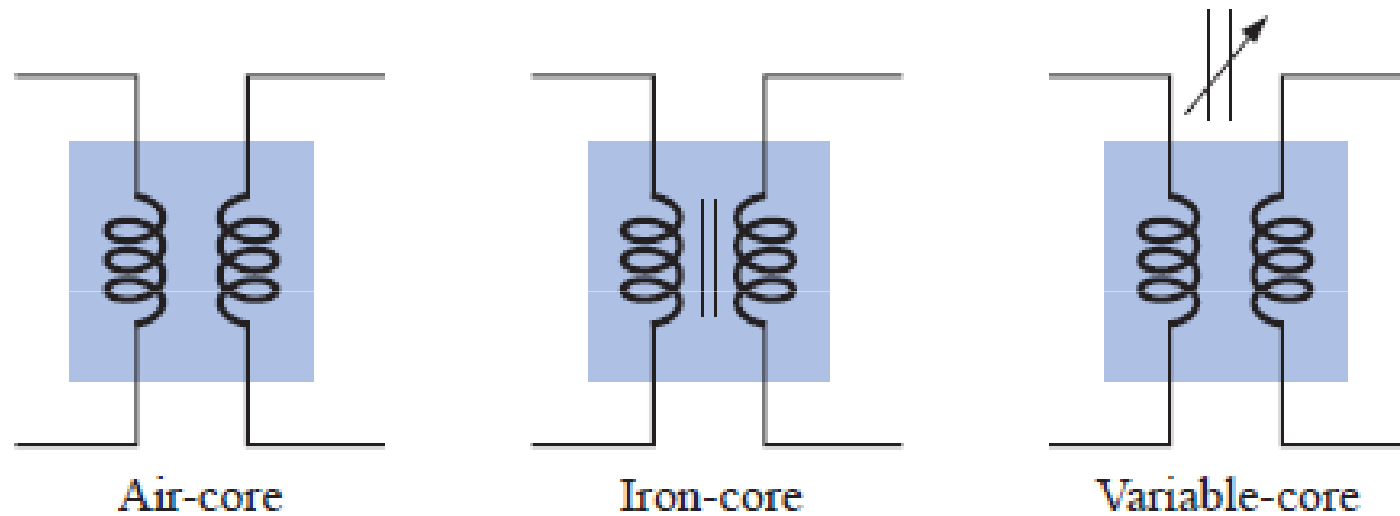
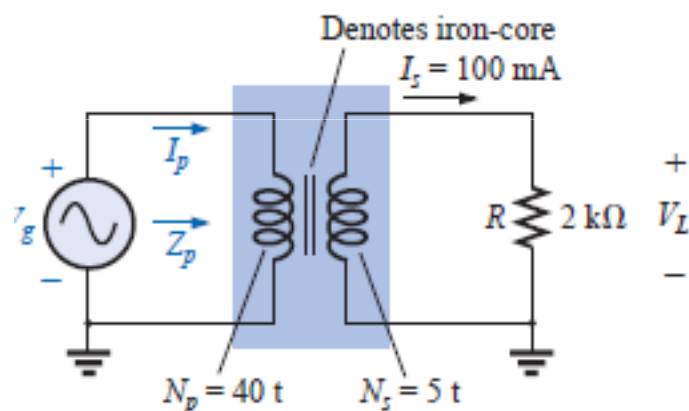


FIG. 21.36

Transformer symbols.

EXAMPLE For the iron-core transformer of Fig. 21.6:

- Find the magnitude of the current in the primary and the impressed voltage across the primary.
- Find the input resistance of the transformer.



$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$I_p = \frac{N_s}{N_p} I_s = \left(\frac{5\text{ t}}{40\text{ t}} \right) (0.1\text{ A}) = 12.5\text{ mA}$$

$$V_L = I_s Z_L = (0.1 \text{ A})(2 \text{ k}\Omega) = 200 \text{ V}$$

$$\text{Also, } \frac{V_g}{V_L} = \frac{N_p}{N_s}$$

$$V_g = \frac{N_p}{N_s} V_L = \left(\frac{40 \text{ t}}{5 \text{ t}} \right) (200 \text{ V}) = \mathbf{1600 \text{ V}}$$

$$Z_p = a^2 Z_L$$

$$a = \frac{N_p}{N_s} = 8$$

$$Z_p = (8)^2 (2 \text{ k}\Omega) = R_p = \mathbf{128 \text{ k}\Omega}$$

Power transfer

- *For the ideal transformer shown, the output power is $P_2 = V_2 \cdot I_2$ w*
- *The input power is $P_1 = V_1 \cdot I_1$ w*
- *Since $V_1 = \frac{N_1}{N_2} \cdot V_2$ and $I_1 = \frac{N_2}{N_1} \cdot I_2$*
- *Then $P_1 = \left(\frac{N_1}{N_2} \cdot V_2\right) \cdot \left(\frac{N_2}{N_1} \cdot I_2\right) = V_2 \cdot I_2 = P_2$*
- *So ideal transformer transfer power with no losses.*

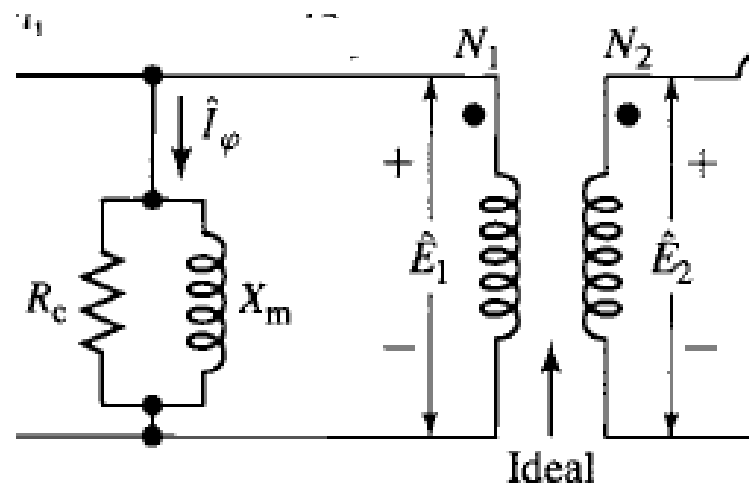
Actual transformers

- *The departure of actual transformers from the ideal one appears in the following:*
- *The magnetic material of the core possesses certain reluctance. This will result in:*
 - *A- Excitation current must flow in the primary winding.*
 - *B- Core losses take place, and current must flow to furnish these losses.*
- *There is leakage flux in both primary, and secondary windings. This will result in voltage drop in these windings proportional to these leakage fluxes.*
- *Transformer windings possess certain resistances. These will result in both copper losses, and voltage drop.*

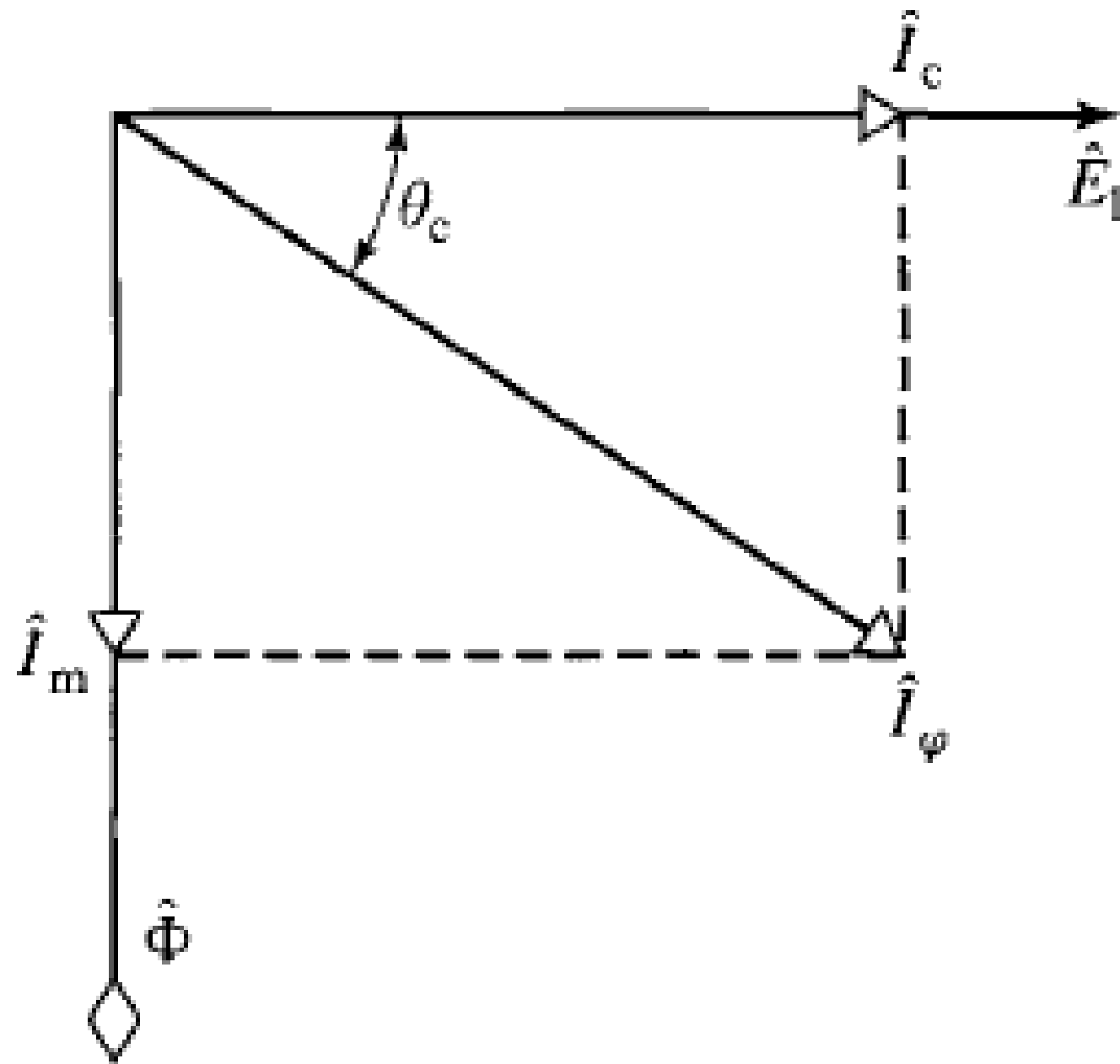
Actual transformer at no-load

- If the primary winding is connected to a sinusoidal voltage source, while its secondary winding is left open, then:*
- 1- The magnetic flux established will be sinusoidal too, with peak value given by:*
- $\phi_{peak} = \frac{E_1}{4.44fN_1}$ wb*
- 2- Induced voltage in secondary is:*
- $E_2 = \frac{N_2}{N_1} \cdot E_1$ volts*

- 3- Current flowing in primary side is small, and consist of two components $\{ I_m , I_c \}$, as shown earlier.
- 4- Then transformer can be represented by an equivalent circuit as:

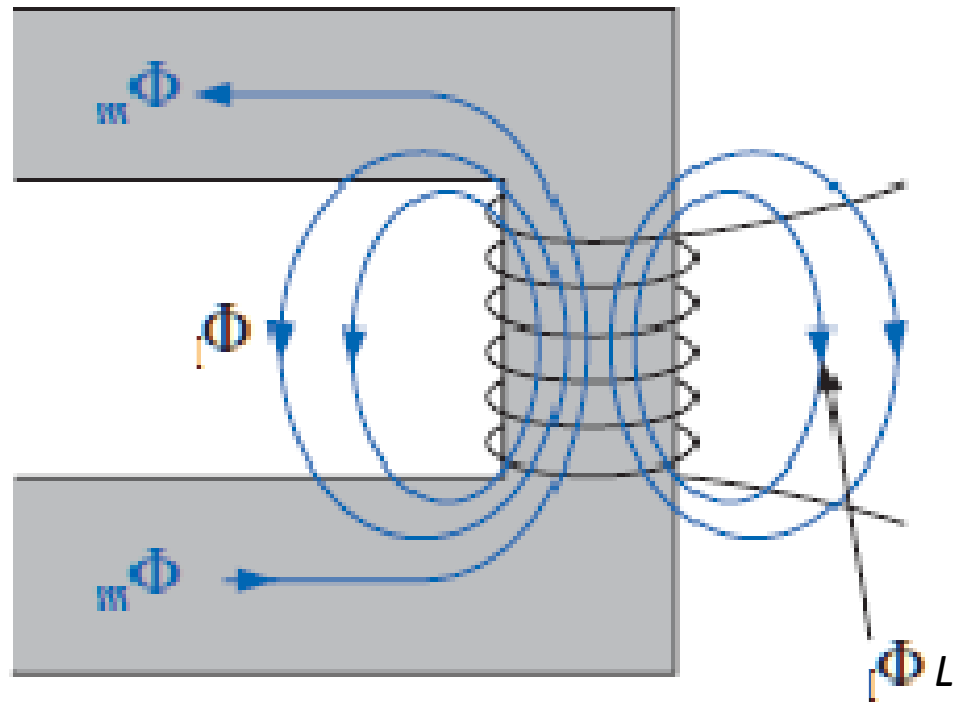


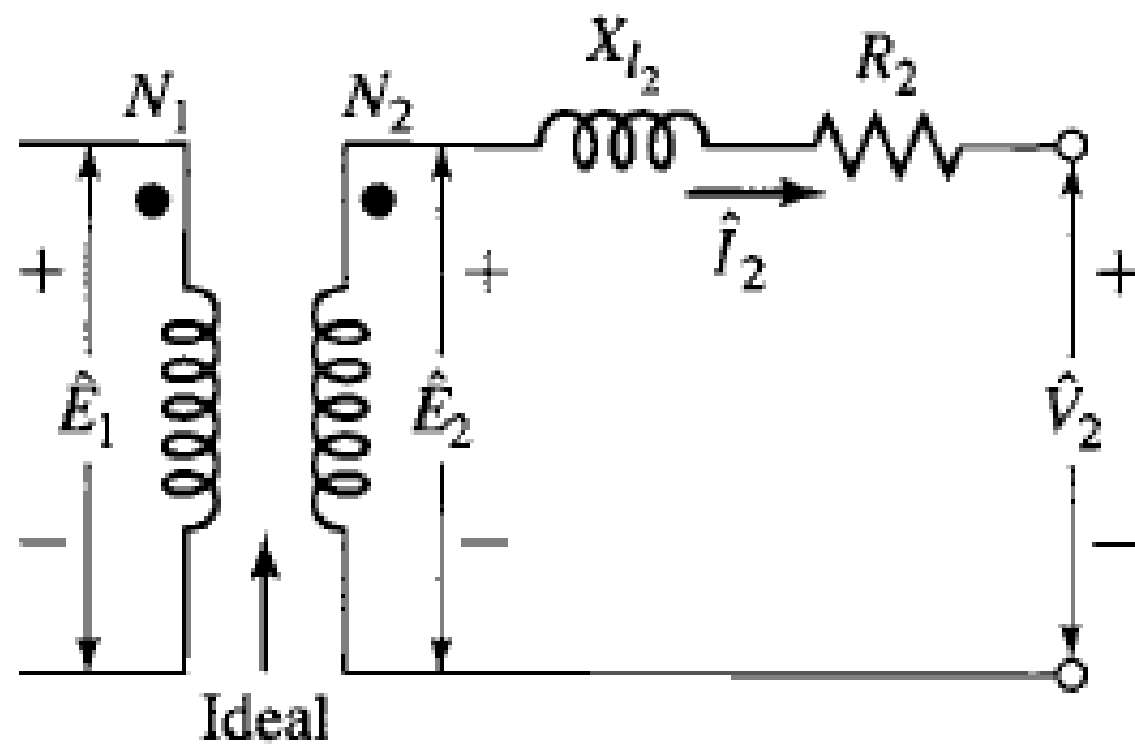
- *Circuit equations are:*
- $I_c = \frac{E_1}{R_c} \text{ A}, \quad I_m = \frac{E_1}{X_m} \text{ A}$
- *Core losses are:* $P_c = E_1 I_0 \cos \theta_c = E_1 I_c \text{ A}$
- *No-load current* $I_0 = \sqrt{I_c^2 + I_m^2} \text{ A}$
- $\theta_c = \tan^{-1} \frac{I_m}{I_c} = \tan^{-1} \frac{R_c}{X_m}$



Actual transformer at load

- *When actual transformer is loaded, a current will flow through the secondary to the load, This current will result in:*
- *1- leakage flux in the secondary winding that will result in voltage drop $\{jI_2X_2\}$.*
- *2- Resistance voltage drop $\{I_2R_2\}$.*
- *Then the internal induced voltage E_2 is:*
- $E_2 = V_2 + I_2(R_2 + jX_2)$





- On the primary side, the primary current I_1 has to:
- 1- Balance the MMF established by secondary current I_2 . As $I_2' N_1 = I_2 I_2$
- 2- Establish the resultant core flux by the resultant primary no-load current I_0 .
- Then $I_1 = I_0 + I_2'$
- This resultant primary current will result in leakage flux, and so voltage drop due to this flux $\{jI_1 X_1\}$. Also a resistance voltage drop will result $\{I_1 R_1\}$.

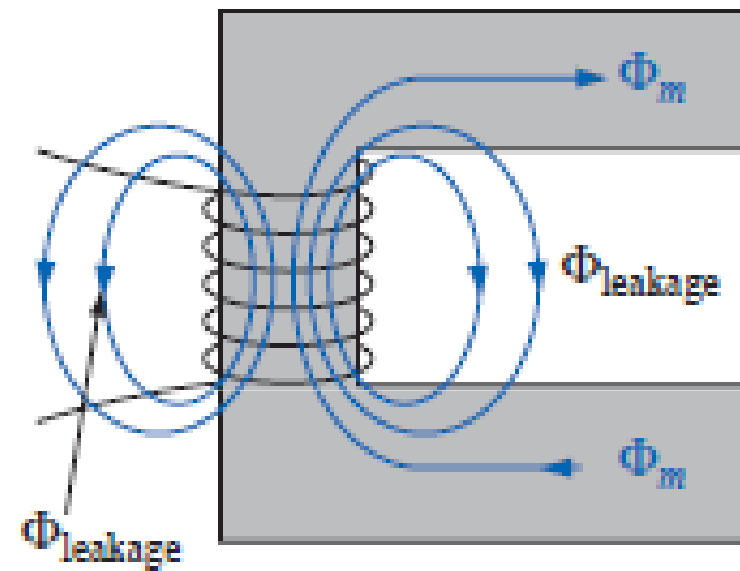
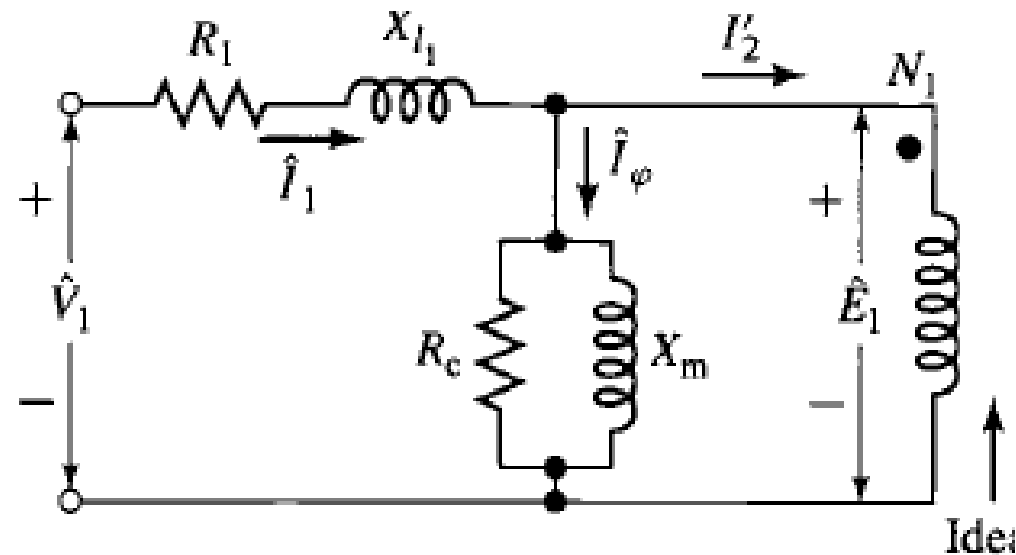
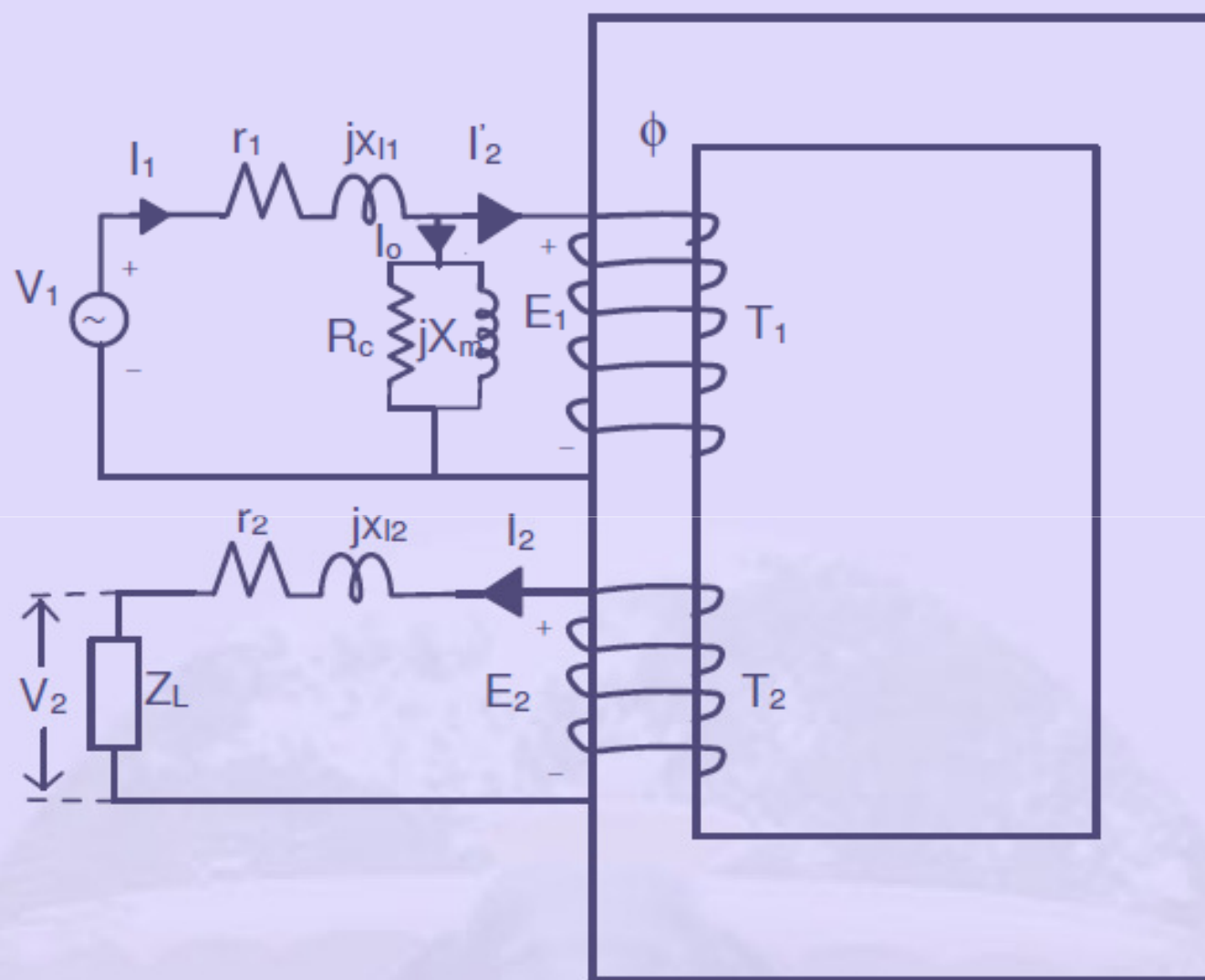


FIG. 21.16

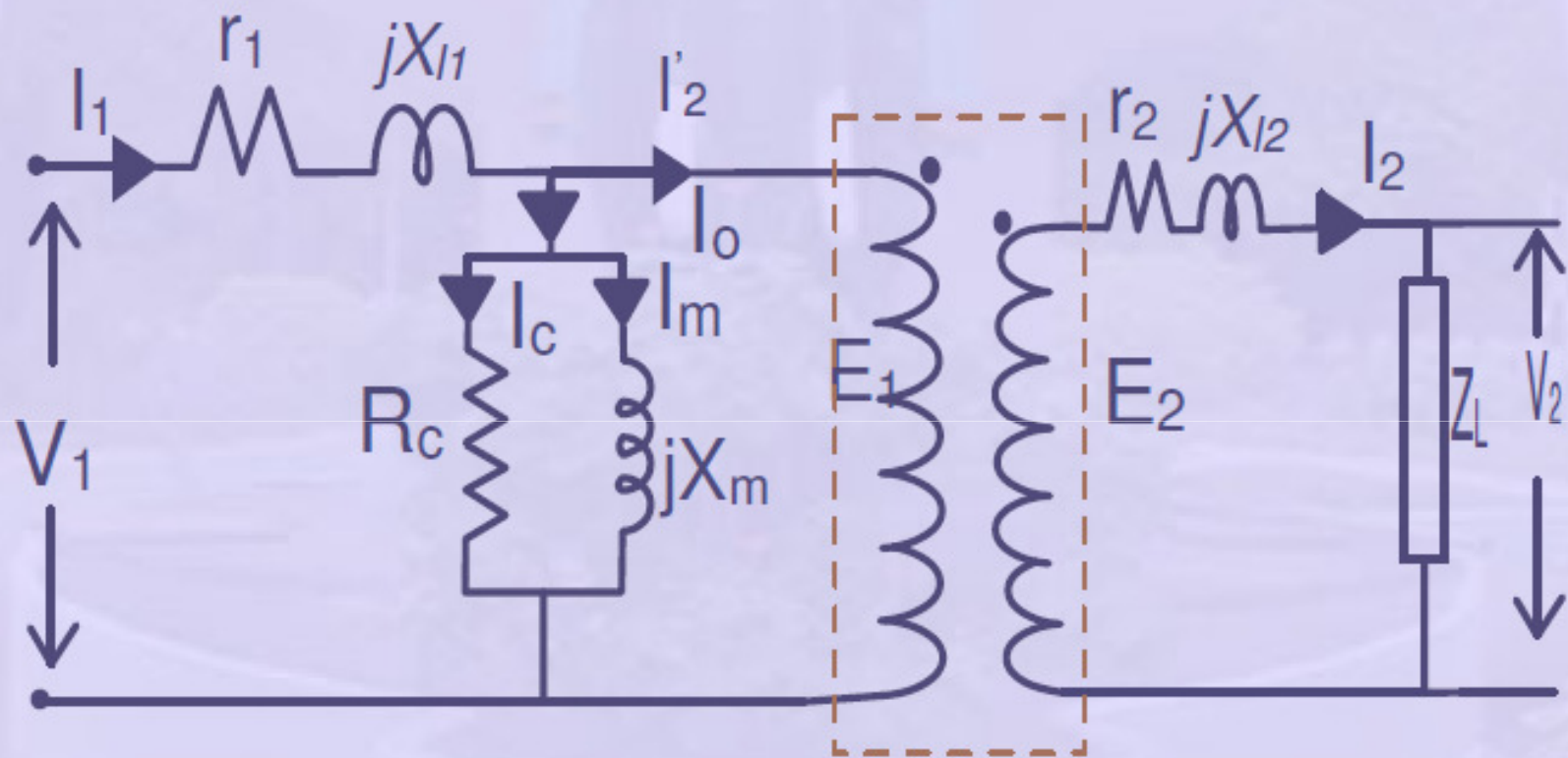
Identifying the leakage flux of the primary.

- *The supply voltage pressed across the primary winding is given by:*
- $V_1 = E_1 + I_1(R_1 + jX_1)$



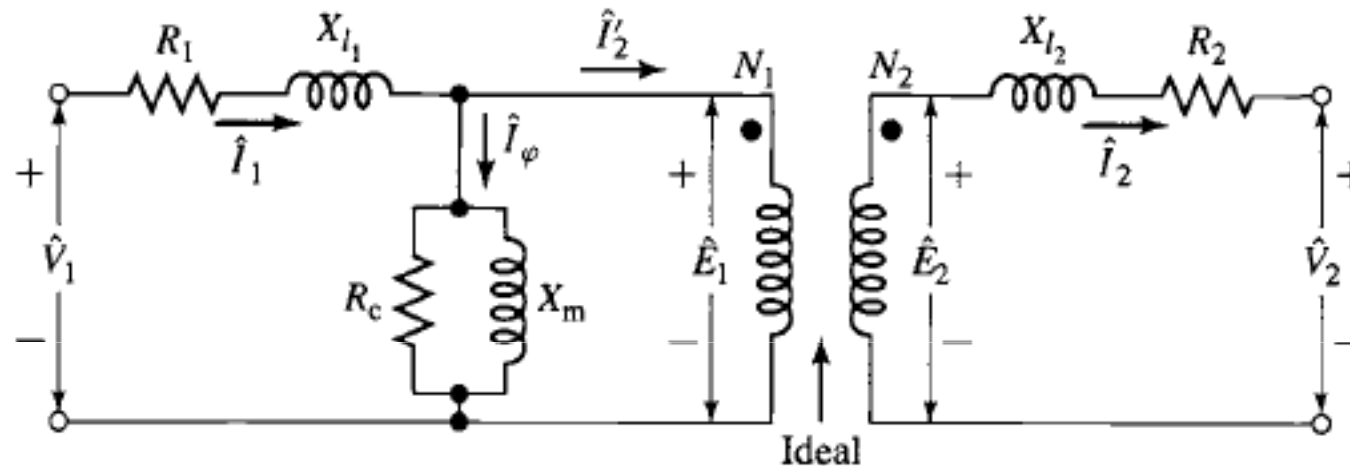


(a) Physical arrangement

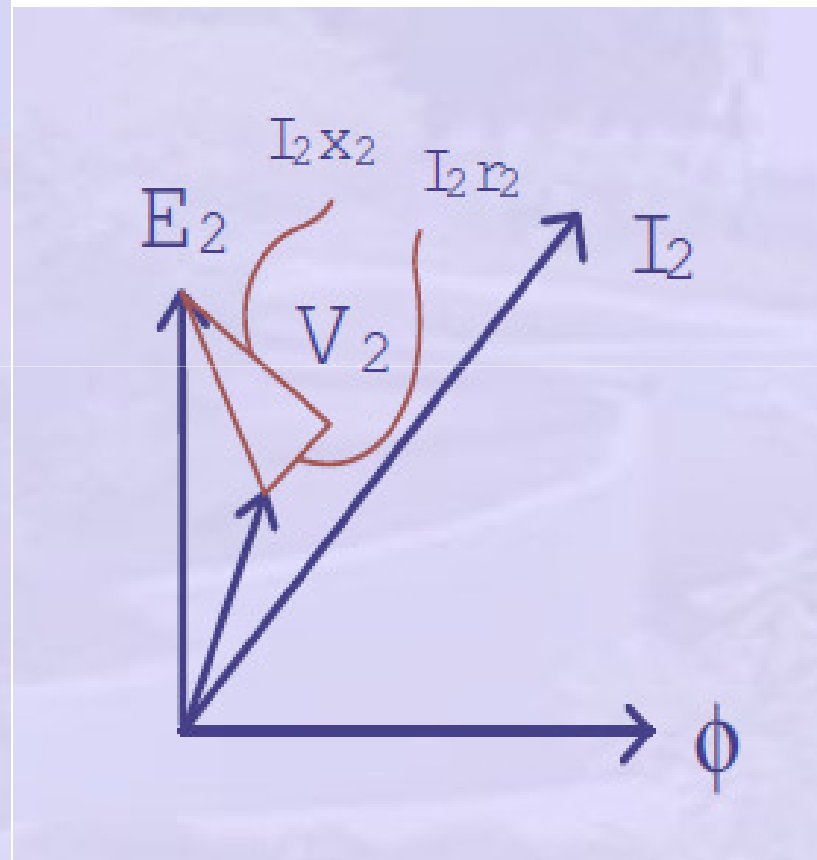
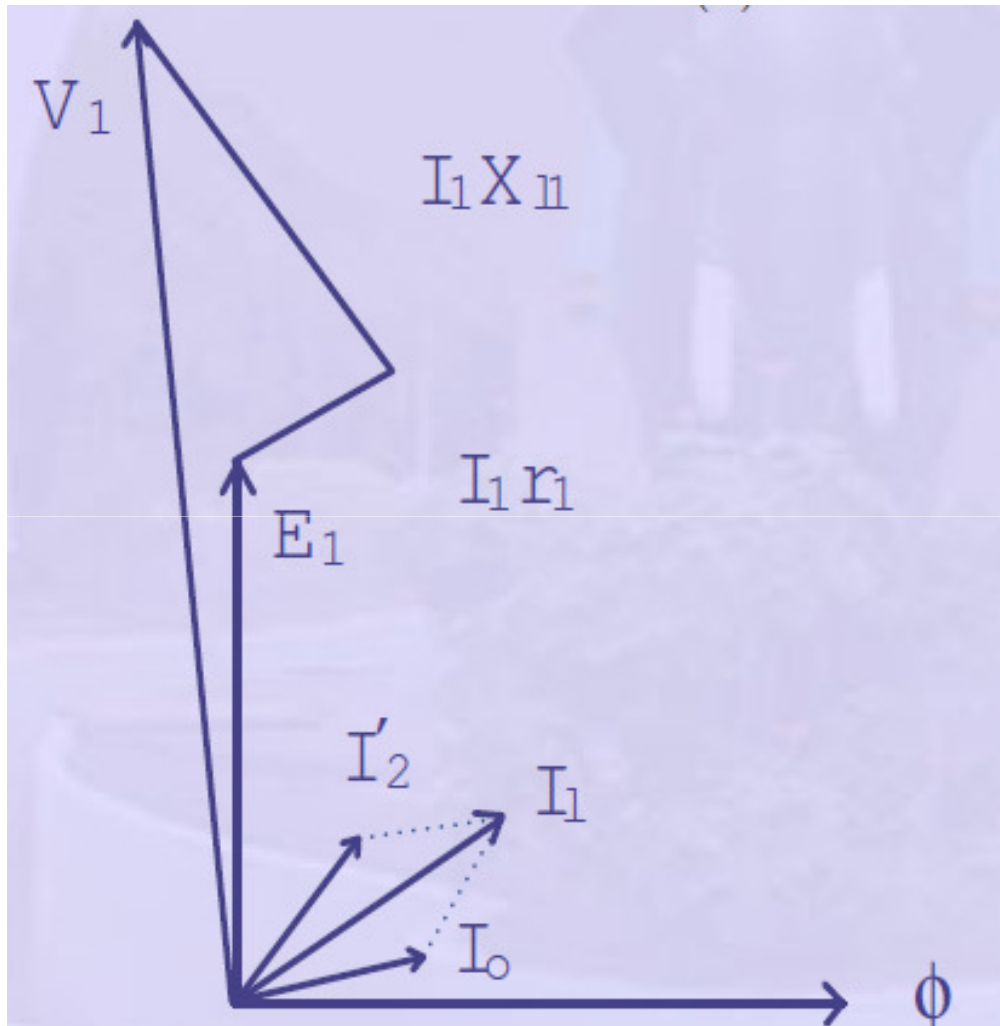


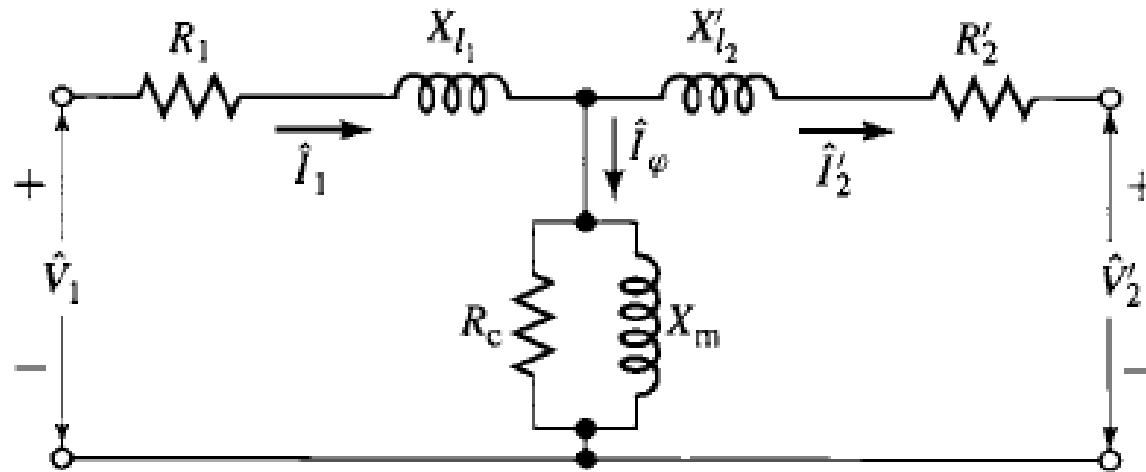
(b) Equivalent circuit

- *So the combined equivalent circuit of practical transformer is:*



- *Referring this equivalent circuit to primary side, we get*





$$R_2' + jX_2' = \left(\frac{N_1}{N_2}\right)^2 (R_2 + jX_2)$$

$$V_2' = \left(\frac{N_1}{N_2}\right) V_2$$

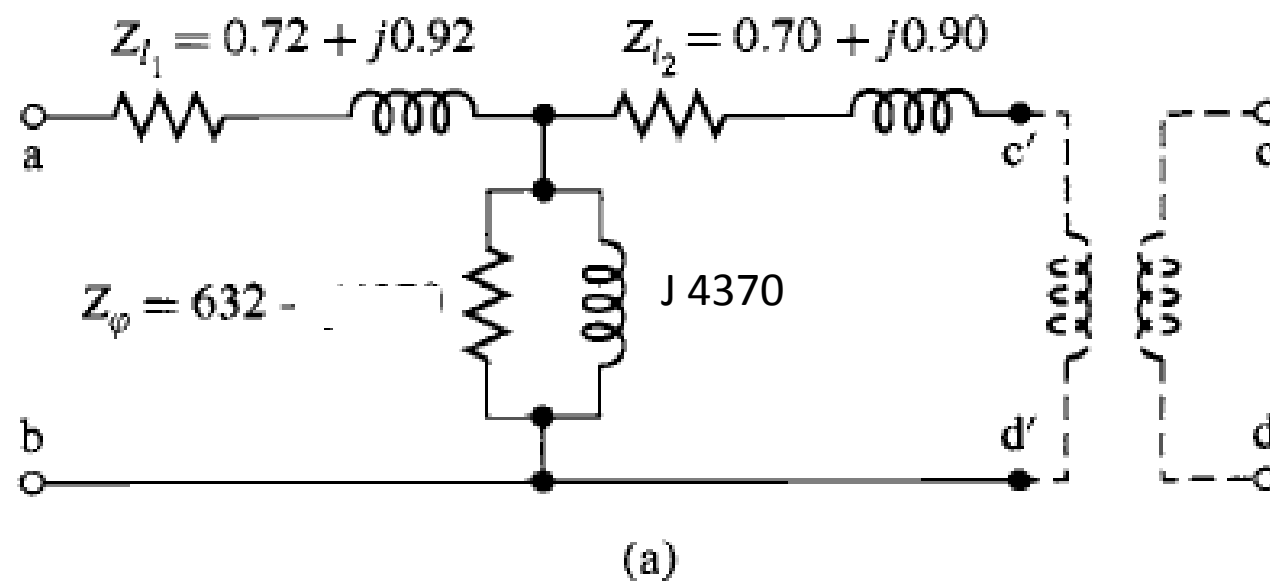
$$I_2' = \left(\frac{N_2}{N_1}\right) I_2$$

Example 1

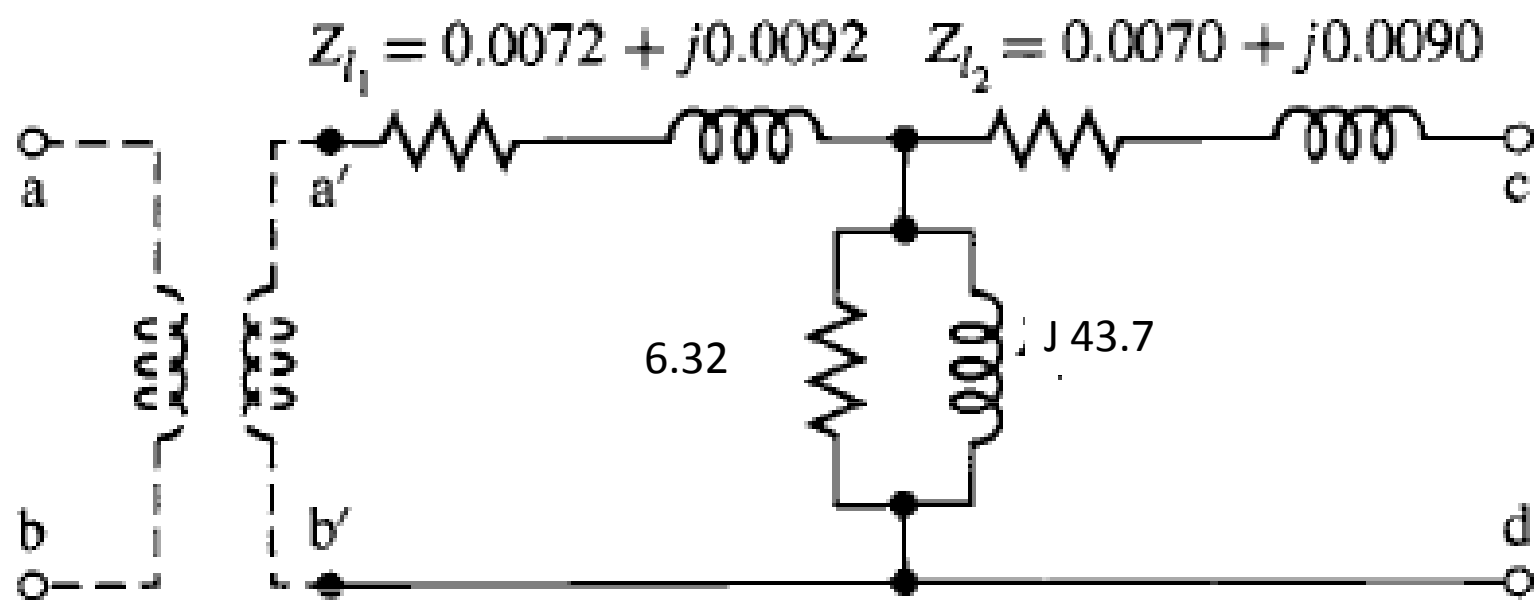
A 50-kVA 2400:240-V 60-Hz distribution transformer has a leakage impedance of $0.72 + j0.92 \, \Omega$ in the high-voltage winding and $0.0070 + j0.0090 \, \Omega$ in the low-voltage winding. At rated voltage and frequency, the impedance Z_ϕ of the shunt branch (equal to the impedance of R_c and jX_m in parallel) accounting for the exciting current is $6.32 \angle j43.7 \, \Omega$ when viewed from the low-voltage side. Draw the equivalent circuit referred to (a) the high-voltage side and (b) the low-voltage side, and label the impedances numerically.

Solution

- By referring the circuit to the primary voltage side, we get:*



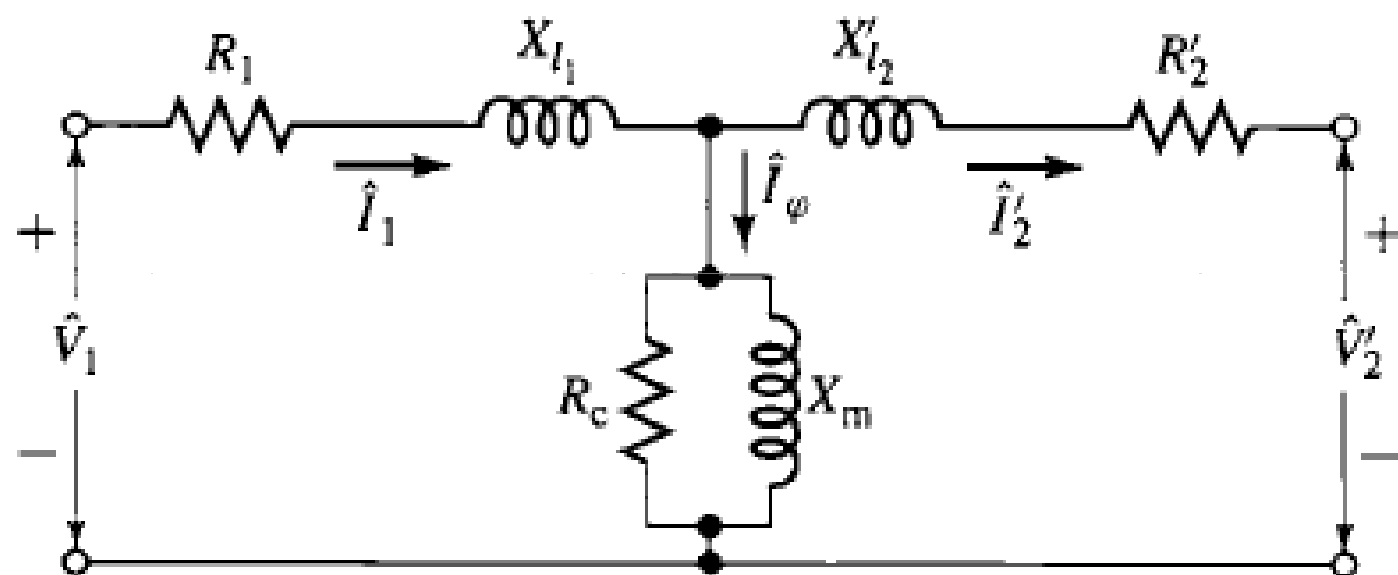
- *Now referring the circuit parameters to the secondary side, we get*



Transformer ratings

- *Transformer ratings depend upon the following:*
- *1- Voltages, which primary and secondary windings can withstand without insulation breakdown or excessive deterioration. These voltages are known as the rated primary and secondary voltages.*
- *2- Currents that can flow in primary and secondary windings without excessive over-heating.*

- *These currents are known as the rated primary and secondary currents. These currents depend upon type of conductor, and method of cooling.*
- *3- The rated apparent power of transformer is obtained by multiplying the rated voltage by the rated current.*
- $S_t = V_1 \cdot I_1 = V_2 \cdot I_2$ VA



- *By referring the equivalent circuit to one side of the transformer, we get:*

- $I'_2 = \frac{S}{V_2} = I'_2 \angle \theta \quad A$

- $E_1 = V'_2 + I'_2 \cdot (R'_2 + jX'_2) \quad V$

- $I_c = \frac{E_1}{R_c} \quad A \quad . \quad I_m = \frac{E_1}{jX_m} \quad A$

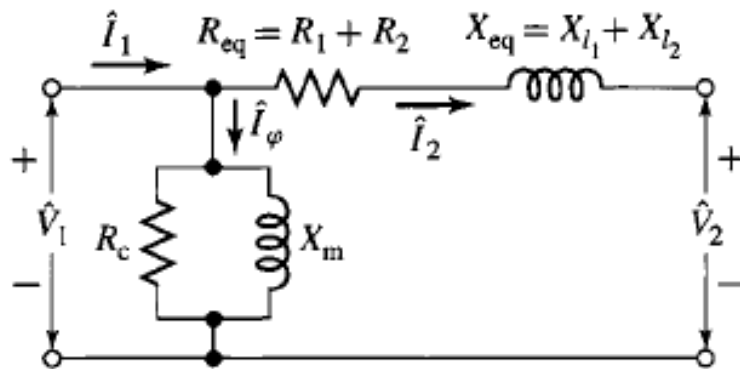
- $I_1 = I'_2 + I_\phi$ A
- $I_\phi = I_c + I_m$ A
- $V_1 = E_1 + I_1 \cdot (R_1 + jX_1)$ V
- *These equations can be represented in a phase diagram as shown:*
- *From this diagram, we get:*
- *Output power P_2 : $P_2 = V'_2 \cdot I'_2 \cdot \cos \theta_2$ W*
- *Input power P_1 : $P_1 = V_1 \cdot I_1 \cos \theta_1$ W*

- *Transformer efficiency :*
- $\eta = \frac{P_2}{P_1} \times 100 \quad \%$
- *Transformer losses :*
- $P_l = P_1 - P_2 \quad W$
- *Voltage regulation of transformer is defined as:*
- $\xi = \frac{V_1 - V'_2}{V'_2} \quad p.u$
- *Draw phase diagram.*

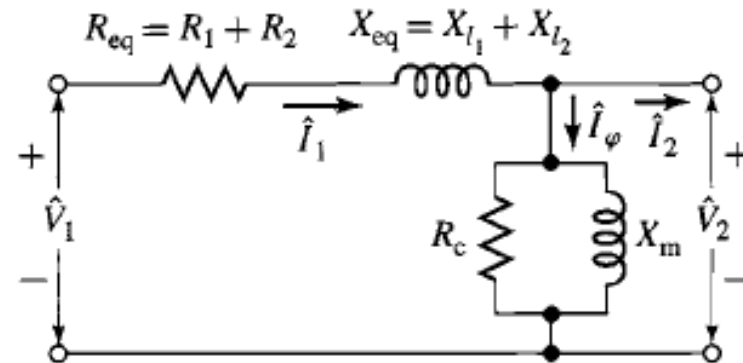
Approximate equivalent circuit

- The iron core of transformer is usually made of magnetic material of good magnetic quality, so the excitation current that will flow at rated transformer voltages is very small { 1-5% of I_{rated} } as compared to rated current of transformer.*
- Also the iron core losses are proportional to the flux density, which is dependent upon transformer voltage. Usually transformer voltage is nearly constant.*

- Then iron core losses can be considered constant, independent of loading of transformer. *So the first approximation* is to move the shunt to either the supply side or load side.



(a)



(b)

- *This approximation allows the combination of primary and secondary impedances together.*

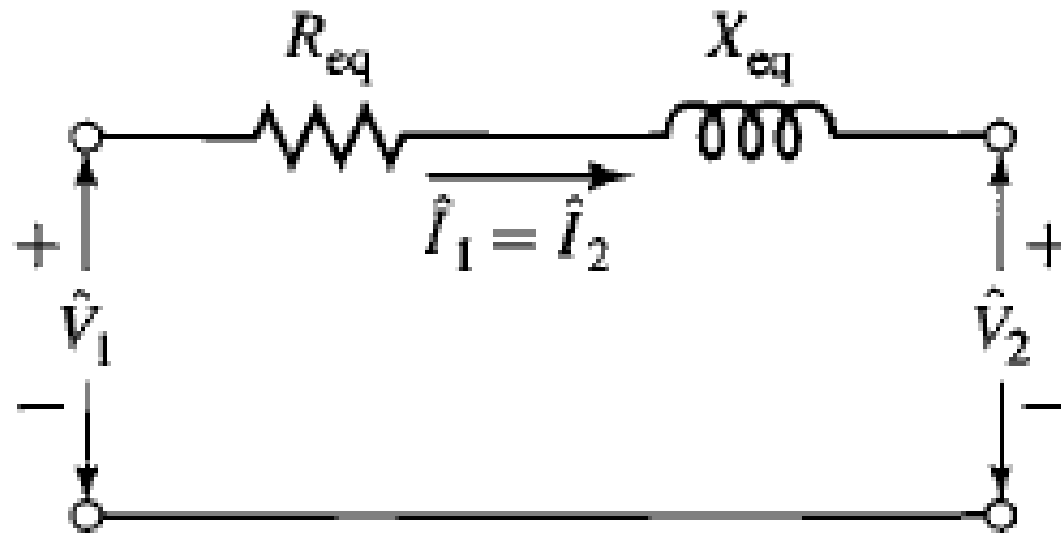
- $Z_{1eq} = Z_1 + \left(\frac{N_1}{N_2}\right)^2 (Z_2) \quad \text{Ohms}$

- $Z_{2eq} = Z_2 + \left(\frac{N_2}{N_1}\right)^2 (Z_1) \quad \text{Ohms}$

- *Where:*

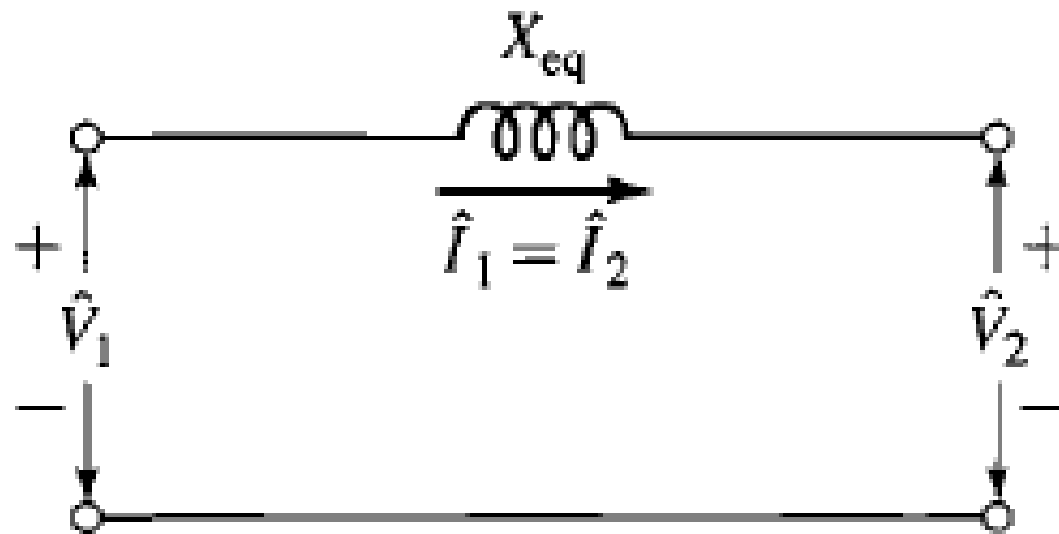
- $Z_1 = R_1 + jX_1 \quad , \quad Z_2 = R_2 + jX_2$

- The second approximation is to neglect the excitation current effect as compared to full load current of transformer. This is done by removing the shunt branch from the equivalent circuit as shown.*



(c)

- The third approximation that is usually done during calculation of short circuit currents. This approximation is based upon the fact that the winding's resistance is very small as compared to winding's leakage reactance.*



(d)

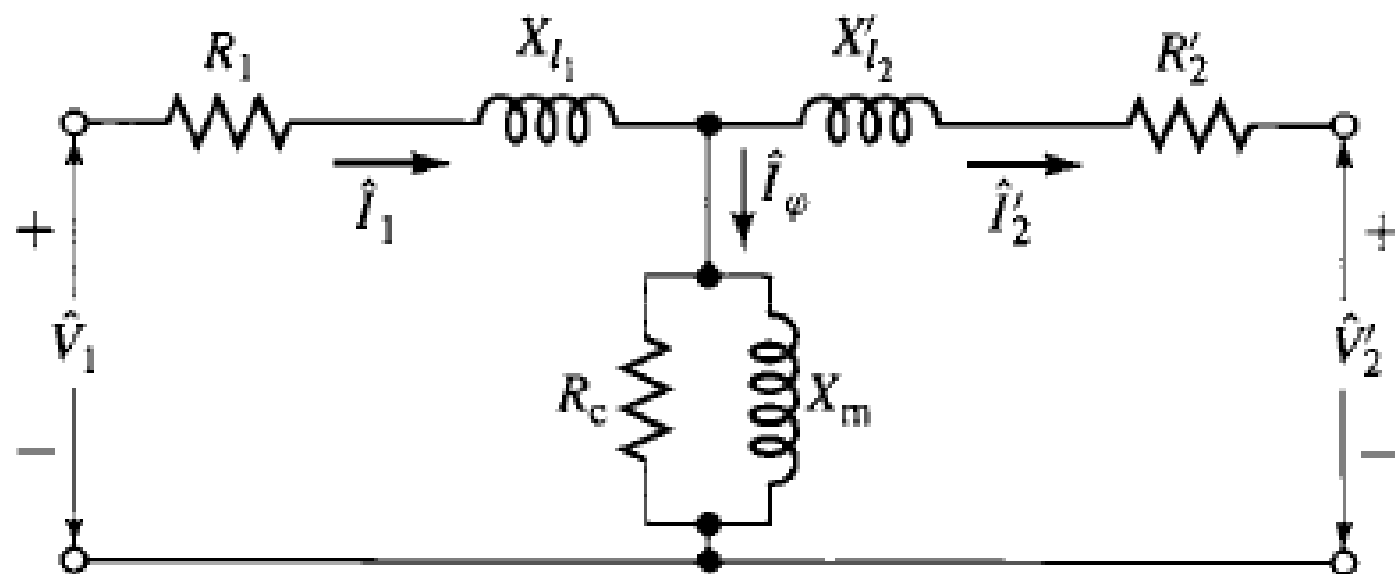
Example

- *A 100 KVA, 11/2.2 KV, 60 Hz single phase transformer has the following equivalent circuit parameters referred to HV side as:*
- $R_1 = 6.1 \quad X_1 = 31.2 \quad \Omega$
- $R_2 = 7.2 \quad X_2 = 31.2 \quad \Omega$
- $R_c = 124 \quad X_m = 57.7 \quad k\Omega$
- *If transformer supplies full load, at 0.8 lag p.f and rated voltage, find:*

- 1- *Using exact equivalent circuit, find: 1- excitation current. 2- Primary supply current. 3- Input power.*
- 2- *Using approximate equivalent circuit, repeat section 1.*
- 3- *compare results obtained in 1 and 2.*

Solution

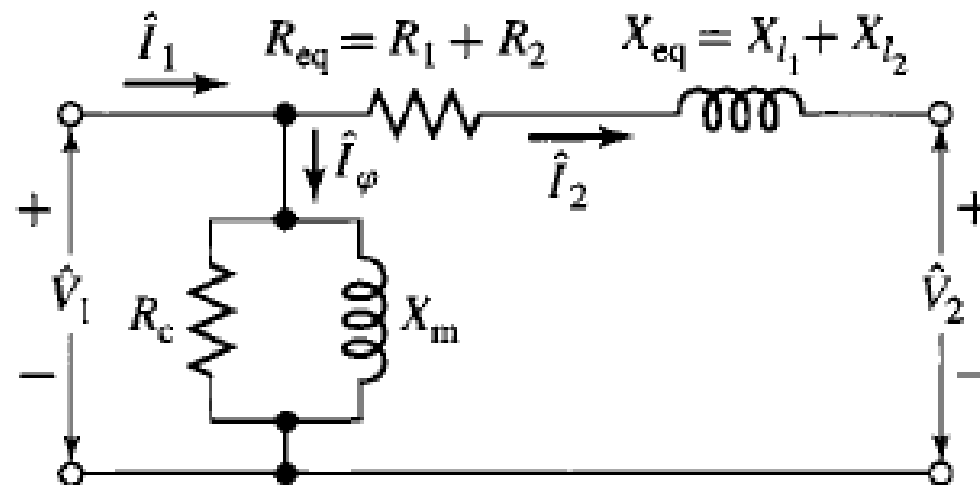
- *Using exact equivalent circuit:*
- $V'_2 = 11 \angle 0 \text{ V}$
- $I'_2 = \frac{100}{11} \angle -\cos^{-1} 0.8 = 9 \angle -36.87 \text{ A}$
- *The shunt branch voltage:*
- $E_1 = V'_2 + I'_2 (Z'_2)$
- $E_1 = 11220.2 + j 185.62 = 11221.74 \angle 0.95$
- *So currents through shunt branch are:*



- $I_c = \frac{E_1}{R_c} = 0.09 \angle 0.95 \quad A$
- $I_m = \frac{E_1}{jX_m} = 0.198 \angle -89.05 \quad A$
- $I_0 = I_c + I_m = 0.2175 \angle -64.6 \quad A$
- *Now supply current { primary current } is:*
- $I_1 = I_0 + I'_2 = 9.1927 \angle -37.5 \quad A$
- *Supply { primary } voltage is:*
- $V_1 = E_1 + I_1(R_1 + jX_1)$
- $V_1 = 11445.5 \angle 1.89 \quad V$

- *The input power is:*
- $P_1 = V_1 \cdot I_1 \cdot \cos \theta_1 \quad \theta_1 = 1.89 - (-37.5)$
- $P_1 = 81.314 \text{ kw}$
- **Now** using approximate equivalent circuit, with shunt branch removed to supply side, then:
- $Z_{1eq} = 63.8 \angle 77.96^\circ \Omega$
- $V_1 = V'_2 + I'_2 \cdot (Z_{1eq})$
- $V_1 = 11438.98 \angle 1.89^\circ \text{ V}$

- *The shunt branch currents are:*
- $I_c = \frac{V_1}{R_c} = 0.0922 \angle 1.89 \text{ A}$
- $I_m = \frac{V_1}{jX_m} = 0.1996 \angle -88.11 \text{ A}$
- $I_0 = 0.22 \angle -63.2 \text{ A}$



- *The supply { primary} current:*
- $I_1 = I_0 + I'_2 = 9.197 \angle -37.48^\circ \text{ A}$
- *The input power:*
- $P_1 = 81.329 \text{ kw}$
- **Note:** *Transformer parameters V_1 , I_1 , P_1 found using approximate equivalent circuit are very close to those found using exact equivalent circuit. So approximate equivalent circuit is usually used in transformer analysis.*

Example

- *For the previous example calculate the % voltage regulation.*

- **Solution:**

- $$\xi = \frac{V_{no\ load} - V_{fload}}{V_{fload}} \times 100 \quad \%$$

- *1- Using exact circuit then:*

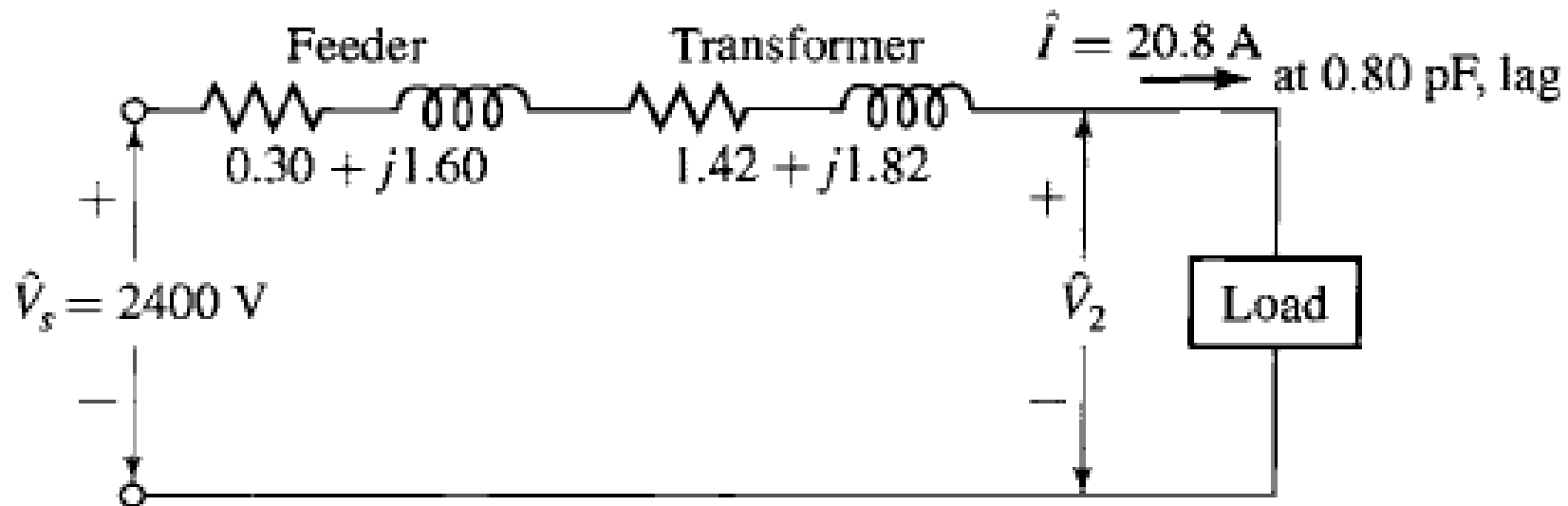
- $$\xi = \frac{11445.5 - 11000}{11000} \times 100 = 4.05 \%$$

- *Using approximate equivalent circuit:*
- $\xi = \frac{11478.98 - 11000}{11000} \times 100 = 3.99 \%$
- **Note** that voltage regulation obtained using approximate equivalent circuit is very close to that obtained from exact equivalent circuit.

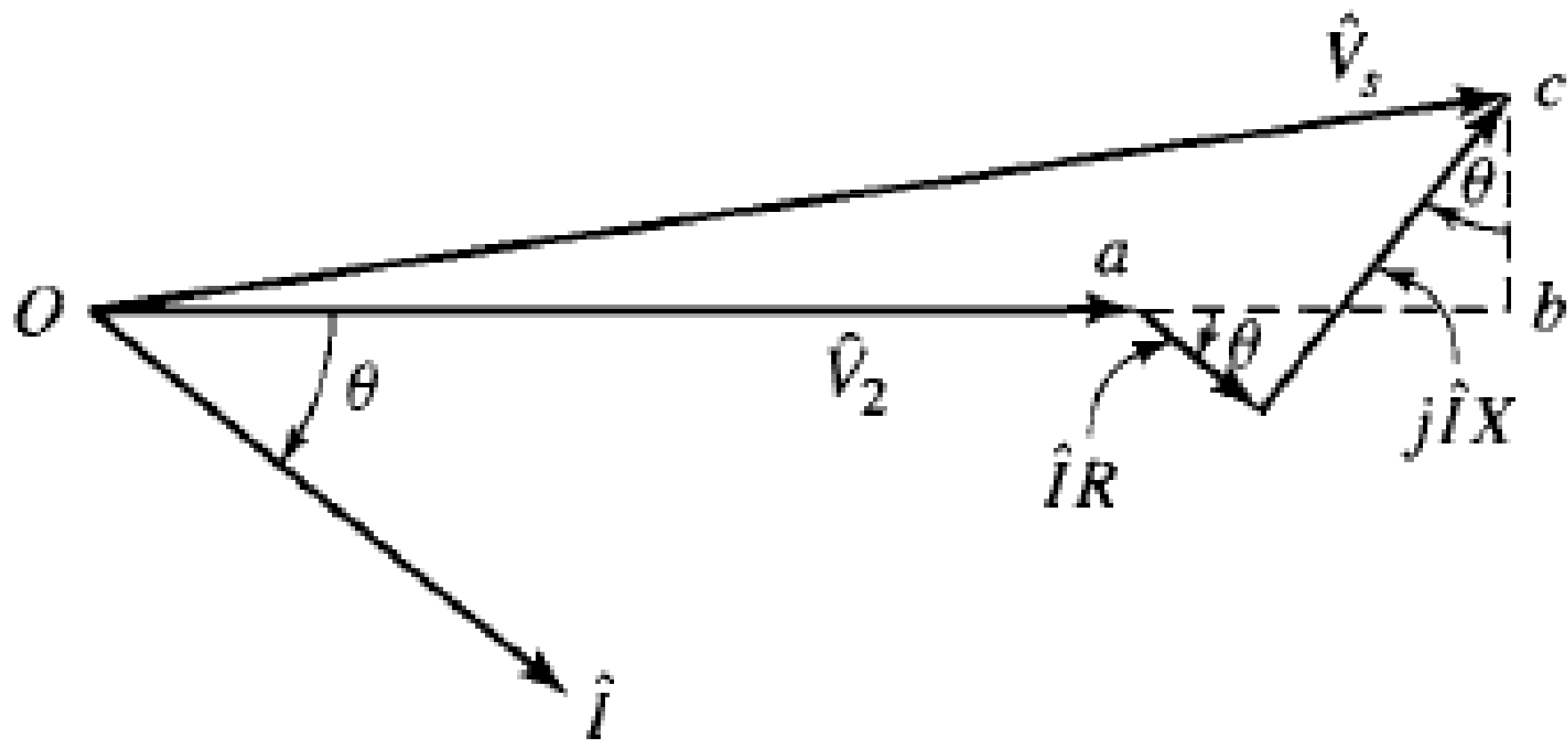
Example

- *A single phase transformer 50 KVA, 2400/240 V, 60 Hz has the approximate equivalent circuit shown below with $Z_{eq} = 1.42 + j1.82$ ohm. The transformer is supplied through a feeder whose impedance is $Z_f = 0.30 + j1.36$ Ohms. If the supply voltage is 2400 V, while the transformer supplies its full-load current, at 0.8 lag. P.f. Find the secondary load voltage, input, and output power.*

- *With all parameters referred to H.V side, we get:*
- $I_{load} = 50 \times 10^3 / 2400 = 20.8 \text{ A}$
- *With V_2 as reference, the angle of current is*



- $V_2 = |V_2| \angle 0$, $I_2 = 20.8 \angle -36.87$ A
- *Drawing the phase diagram, we get:*
-



- *With $V_s = 2400$ V, the load voltage V_2 can be found using trigonometric analysis as:*

$$Ob = \sqrt{V_s^2 - (bc)^2} \quad \text{and} \quad V_2 = Ob - ab$$

$$bc = IX \cos \theta - IR \sin \theta \quad ab = IR \cos \theta + IX \sin \theta$$

$$bc = 20.8(3.42)(0.80) - 20.8(1.72)(0.60) = 35.5 \text{ V}$$

$$ab = 20.8(1.72)(0.80) + 20.8(3.42)(0.60) = 71.4 \text{ V}$$

Substitution of numerical values shows that $V_2 = 2329$ V, referred to the high-voltage

$$V_2 = 233 \text{ V}$$

- *The output power P_2 is:*
- *$P_2 = 50 \times 0.8 = 40 \text{ KW}$*
- *The input power P_1 is:*
- *$P_1 = P_2 + (I_2)^2 \cdot (0.3 + 1.4) = 40.735 \text{ KW}$*

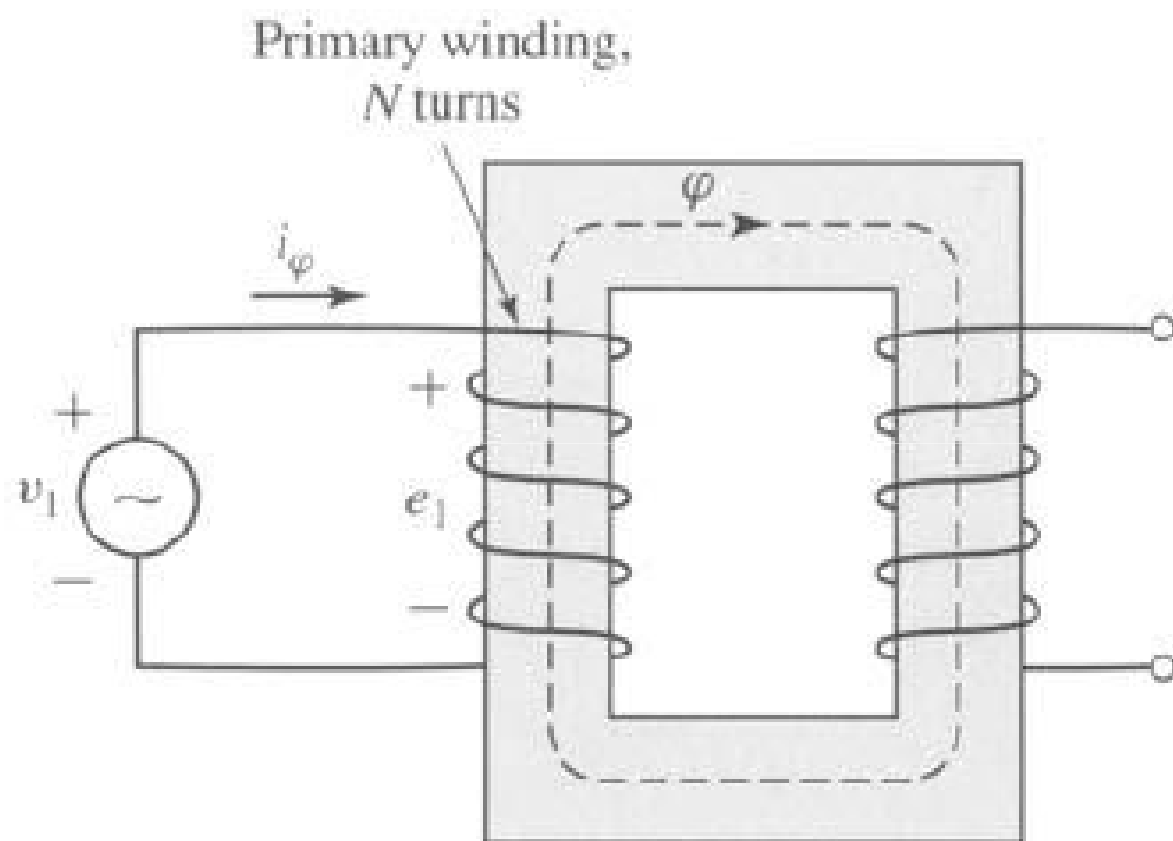
Transformer parameters by test

- *The main transformer parameters are:*
- *1- The equivalent circuit.*
- *2- Transformer losses.*
- *3- Transformer voltage regulation.*
- *There are two experiments that can be done on transformer to obtain all previous parameters. These experiments are:*
- *No load test, and short circuit test.*

No-load test

- *With one side of the transformer left open, while the other side is supplied by variable ac voltage until rated voltage is applied across its terminals, then we note that:*
- *The current taken by the transformer is very small as compared to its rated current { less than 5% of rated current}.*
- *Then the copper losses produced by this current is very small, and so all input power is consumed as core losses.*

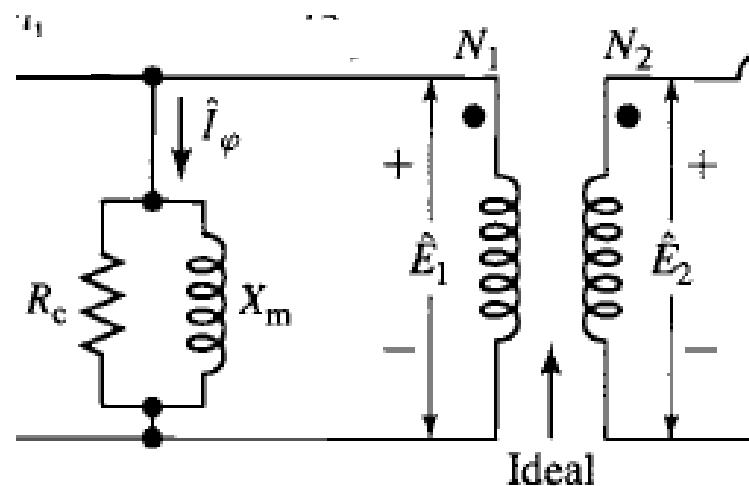
- *Also the leakage flux produced by this current is very small too, and so the voltage drop in the primary side is small and can be neglected.*
- *Then the measured input power at rated voltage is equal to the iron core losses of the transformer.*
- *Also the equivalent circuit at no-load is as shown.*



- *The parameters of the equivalent circuit is:*

$$R_c = \frac{V_{oc}^2}{P_{oc}} \qquad |Z_\phi| = \frac{V_{oc}}{Z_{oc}}$$

$$X_m = \frac{1}{\sqrt{(1/|Z_\phi|)^2 - (1/R_c)^2}}$$



$$W_0 = V_1 I_c = P_{core}$$

$$\therefore I_c = \frac{W_0}{V_1}$$

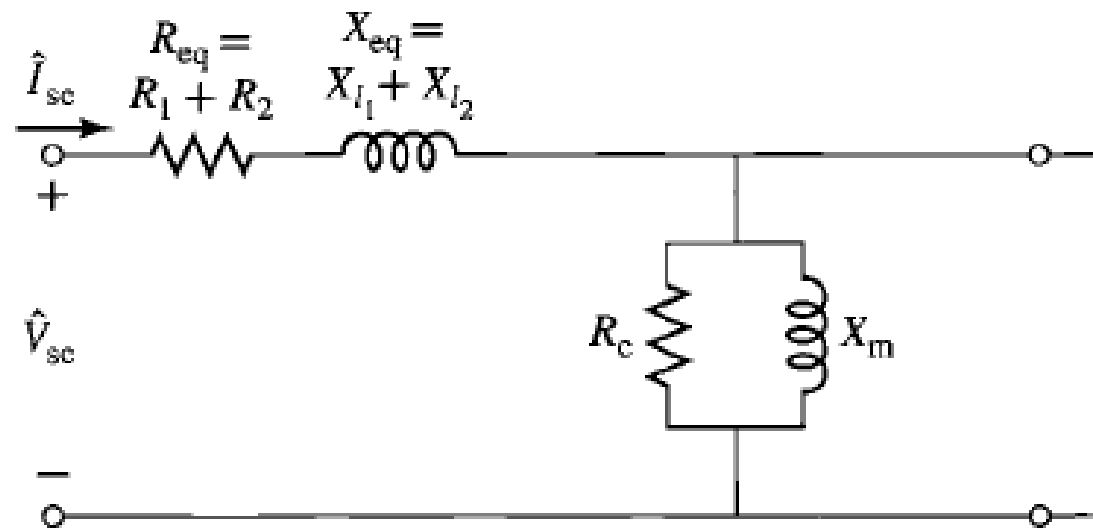
$$\therefore I_m = \sqrt{I_0^2 - I_c^2} \quad \text{or}$$

$$R_c = \frac{V_1}{I_c} \quad \text{and} \quad X_m = \frac{V_1}{I_m}$$

Short circuit test

- *With one side of the transformer short circuited, while the other side is supplied by a very low variable ac voltage, change the supply voltage slowly until rated current flows through the transformer. Measure the input power and supply voltage. Then we note that:*
- *1- The applied voltage needed to circulate rated current through transformer is very small { less than 5 % of rated current}. So the core flux is also very small, and the excitation current needed can be neglected.*

- Also the corresponding core losses are very small, and all input power can be assumed to be consumed as winding copper losses inside the transformer.
- Then the apparent equivalent circuit is as shown.



- *The equivalent circuit parameters are as shown:*

$$|Z_{\text{eq}}| = |Z_{\text{sc}}| = \frac{V_{\text{sc}}}{I_{\text{sc}}}$$

$$R_{\text{eq}} = R_{\text{sc}} = \frac{P_{\text{sc}}}{I_{\text{sc}}^2}$$

$$X_{\text{eq}} = X_{\text{sc}} = \sqrt{|Z_{\text{sc}}|^2 - R_{\text{sc}}^2}$$

Combined equivalent circuit

- *By combining the equivalent circuits obtained from previous two tests, we get the overall equivalent circuit of transformer.*

Example

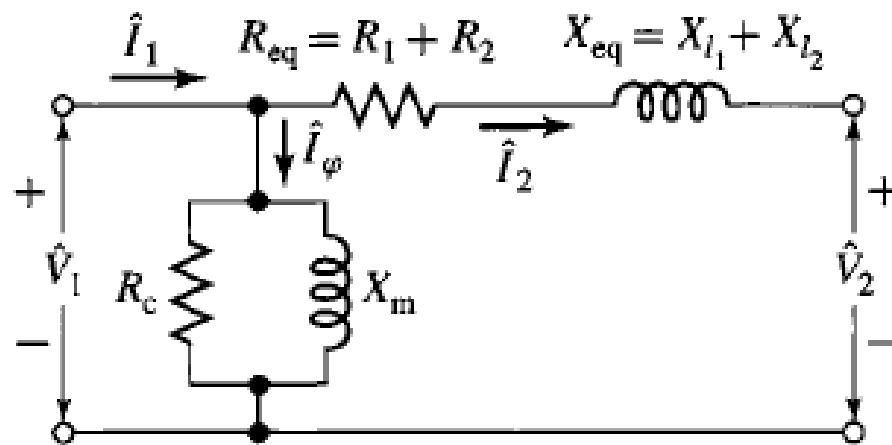
- *A 12 KVA, 230/400 V, 50 Hz single phase transformer gave the following data:*
- *No-load test: 230 V, 2 A, 120 W.*
- *Short-circuit test: 25 V, 25 A, 80 W. Find:*
- *1- Equivalent circuit referred to 400 V side.*
- *2- Efficiency at full load, 0.8 lag. P.f*
- *3- Efficiency at 50% load, 0.8 lag. P.f*

Solution

- *From transformer ratings, we calculate the transformer rated current on both sides as:*
- $I_1 = \frac{12 \times 10^3}{400} = 30 \text{ A} , I_2 = \frac{12 \times 10^3}{230} = 52.17 \text{ A}$
- *From no-load test:*
- $P_0 = V I_0 \cos \varphi_0$
- $\cos \varphi_0 = \frac{120}{230 \times 2} = 0.26 \quad \sin \varphi_0 = 0.96$
- *The excitation current components are:*

- $I_c = I_0 \cos \varphi_0 = 0.52 \text{ A}$
- $I_m = I_0 \sin \varphi_0 = 1.93 \text{ A}$
- *Then:*
- $R_c = \frac{V_1}{I_c} = 442.3 \Omega$, $X_m = \frac{V_1}{I_m} = 11.9 \Omega$
- *Now refer this circuit to 400 V side as:*
- $R'_c = \left(\frac{400}{230}\right)^2 \times 442.3 = 1337.8 \Omega$
- $X'_m = 360 \Omega$
- *From short-circuit test:*

- $R_{eq} = \frac{P_{sc}}{I_{sc}^2} = 0.128 \Omega$
- $Z_{eq} = \frac{V_{sc}}{I_{sc}} = \frac{25}{25} = 1 \Omega$
- $X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = 0.99 \Omega$
- *The combined equivalent circuit is:*



- *Copper losses at full-load is:*
- $P_{cu f.l} = I_{f.i}^2 \cdot R_{eq} = 30^2 \times 0.128 = 145.2 \text{ W}$
- *The transformer efficiency is:*
- $$\eta = \frac{\text{output power}}{\text{input power}}$$

$$= \frac{\text{output}}{\text{output} + \text{core losses} + \text{copper losses}}$$

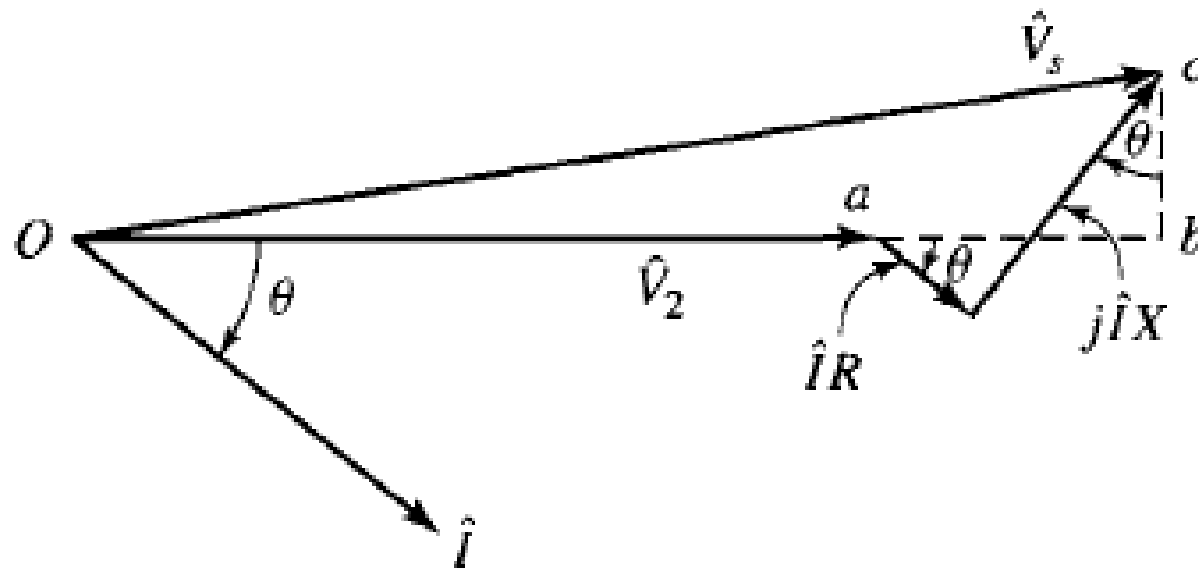
$$= \frac{12 \times 0.8 \times 1000}{12 \times 0.8 \times 1000 + 120 + 145.2} = 0.97 = 97\%$$
- *At 50% loading:*
- $$\eta = \frac{0.5 \times 12000 \times 0.8}{0.5 \times 12000 \times 0.8 + 120 + 0.5^2 \times 145.2} = 96.85\%$$

Voltage regulation

- *We can obtain an approximate equation for voltage regulation using the approximate equivalent circuit of transformer as:*
- *With all parameters referred to one side of transformer, the following phase diagram can be obtained:*

- $$\xi = \frac{|V'_s - V'_2|}{|V'_2|} \triangleq \frac{ab}{oa}$$

$$= (IR_{eq} \cos \theta + IX_{eq} \sin \theta) / V'_2 \quad p.u$$



- *In general, we can obtain the following equation:*

- $$\xi = \frac{I(R_{eq} \cos \theta \pm X_{eq} \sin \theta)}{|V|} \times 100 \%$$

- *(+) for lagging power factor.*

- *(-) for leading power factor.*

- *Also transformer efficiency can be written as:*

- $$\eta = \frac{nS_r \cos \theta}{nS_r \cos \theta + P_0 + n^2 p_{cu}} \times 100 \%$$

- *Where:*

- S_r = rated power of transformer VA
- P_{cu} = full load copper losses of transformer W
- P_o = core losses of transformer W
- $\cos \vartheta$ = load power factor.
- **Example:** for the previous example calculate the transformer voltage regulation at full load and half load, and 0.8 lag. p.f.
- A: at full load:
 - $\xi = \frac{30(0.128 \times 0.8 + 0.99 \times 0.6)}{400} \times 100 = 5.22 \%$
- B: at half load:
 - $\xi = \frac{15(0.128 \times 0.8 + 0.99 \times 0.6)}{400} \times 100 = 2.61 \%$

- **Maximum** efficiency of transformer occurs when the transformer copper losses are exactly equal to iron core losses. This occurs at a loading ratio n given by:
- $n^2 P_{cu} = P_0$
- $n = \sqrt{\frac{P_0}{P_{cu}}} \quad p.u$

Home work # 4

- *Auto-transformer:*
- *In normal two winding transformers, the windings are electrically isolated from each other, while they are mutually coupled by the coupling magnetic flux. Power can flow from one side to the other only through this coupling magnetic flux.*
- *In auto-transformers there is a direct electrical connection between transformer windings.*

- *So in auto-transformers, power can flow by the coupling magnetic flux as well as through the direct electrical connection.*
- *Auto-transformers are more efficient, smaller in size, and cost less than an equivalent two winding transformer of the same rating.*
- *On the other hand auto-transformers has smaller leakage reactance, and larger fault current can occur as compared to normal two winding transformer of the same rating.*

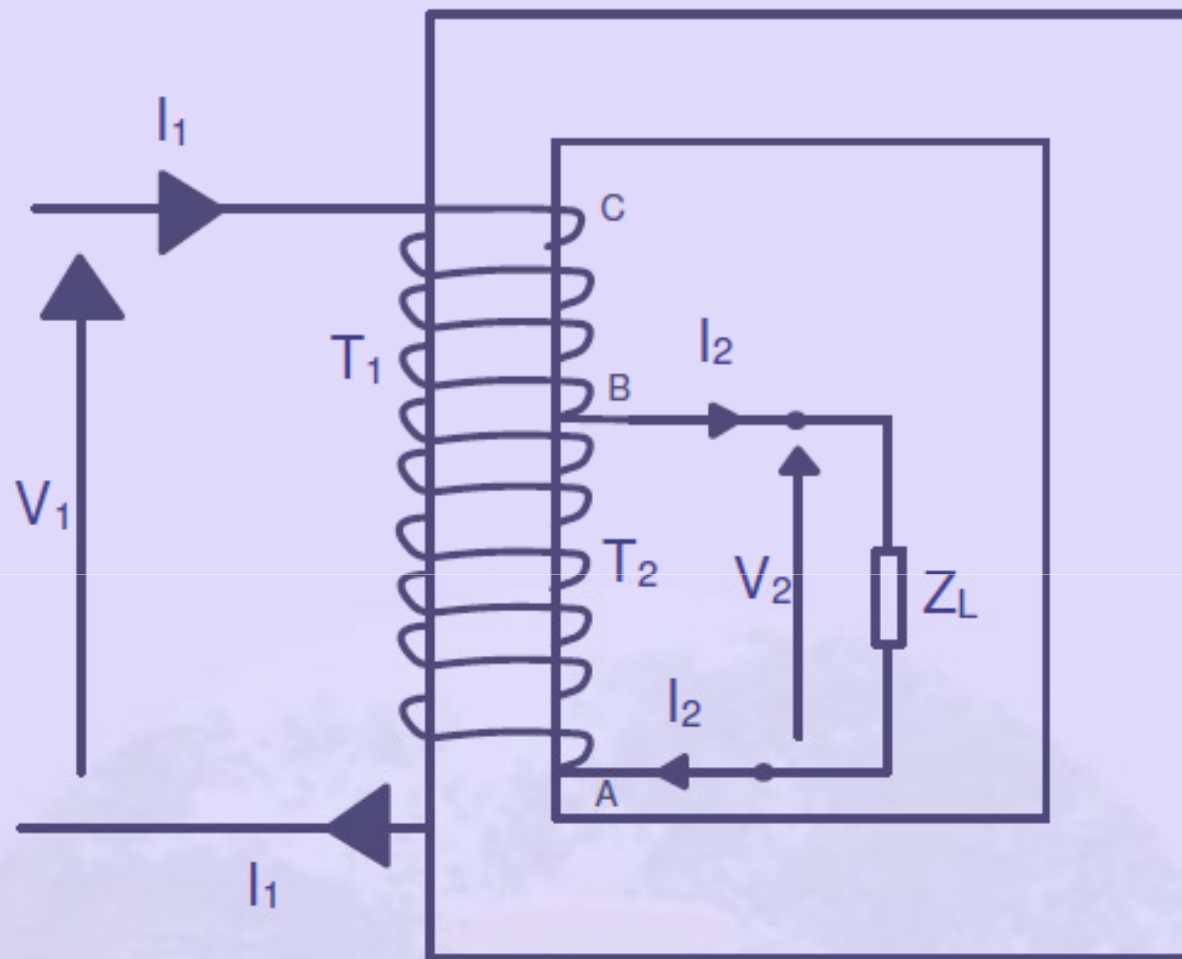
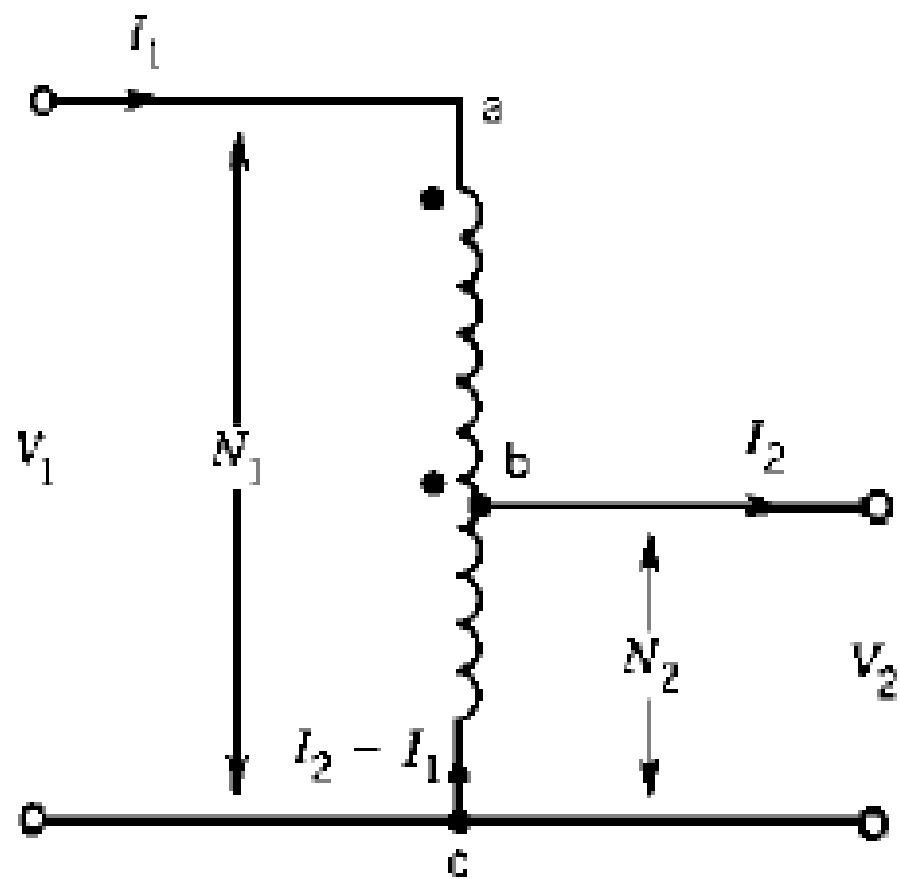


Figure 28: Autotransformer - Physical Arrangement



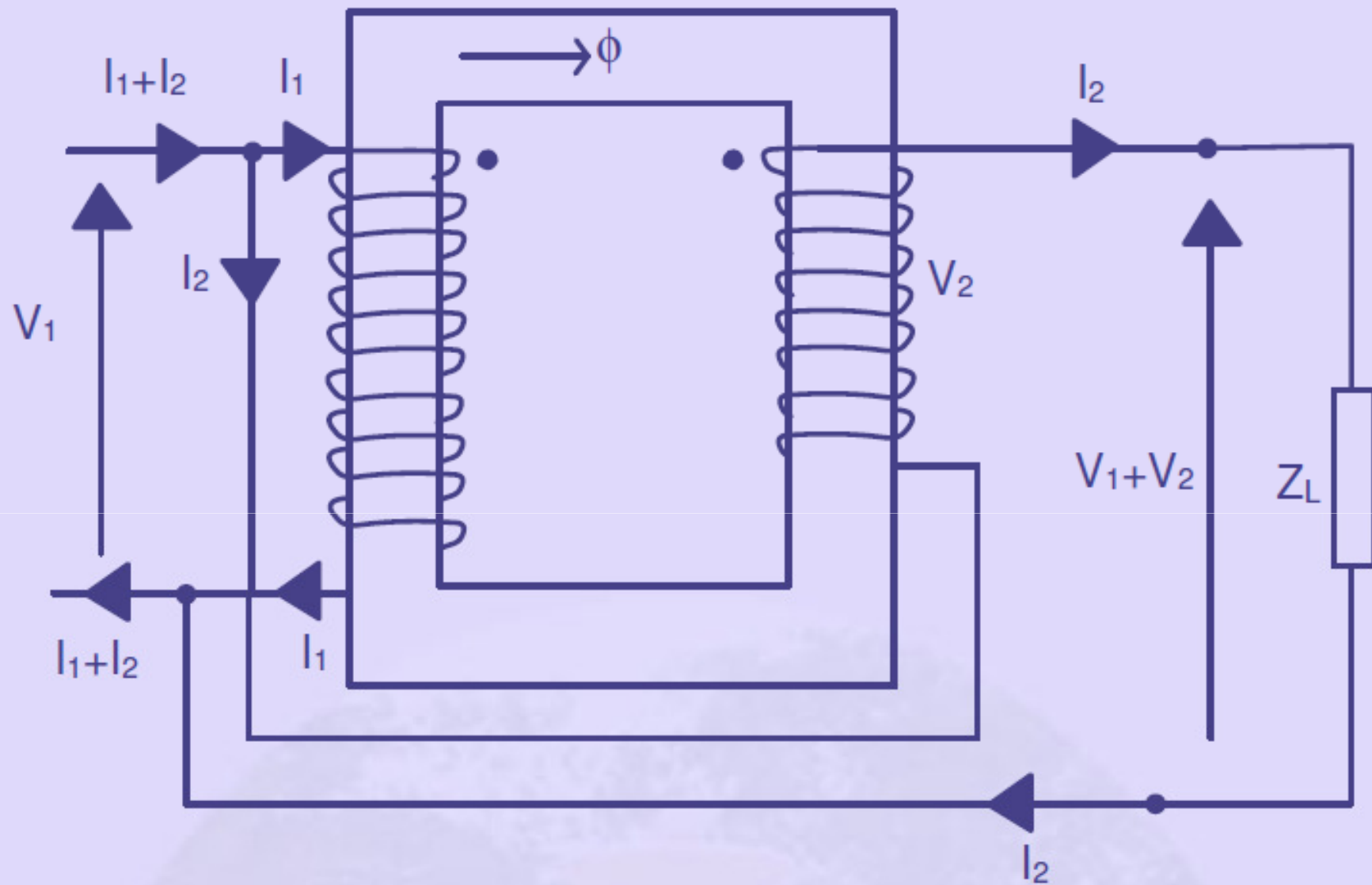


Figure 29: Two Winding Transformer used as auto transformer

Advantages of Auto-transformer

- It effects a saving in winding material (copper or aluminum), since the secondary winding is part of the primary current.
- Lower copper loss, therefore efficiency is higher than in the two winding transformer.
- Lower leakage reactances, lower exciting current.
- Variable output voltage can be obtained.

Disadvantages of Auto-Transformer

- There is a direct connection between the primary and secondary sides.
- Should an open-circuit develop between points **b and c**, the full mains voltage would be applied to the secondary.
- The short-circuit current is much larger than for normal two-winding transformer

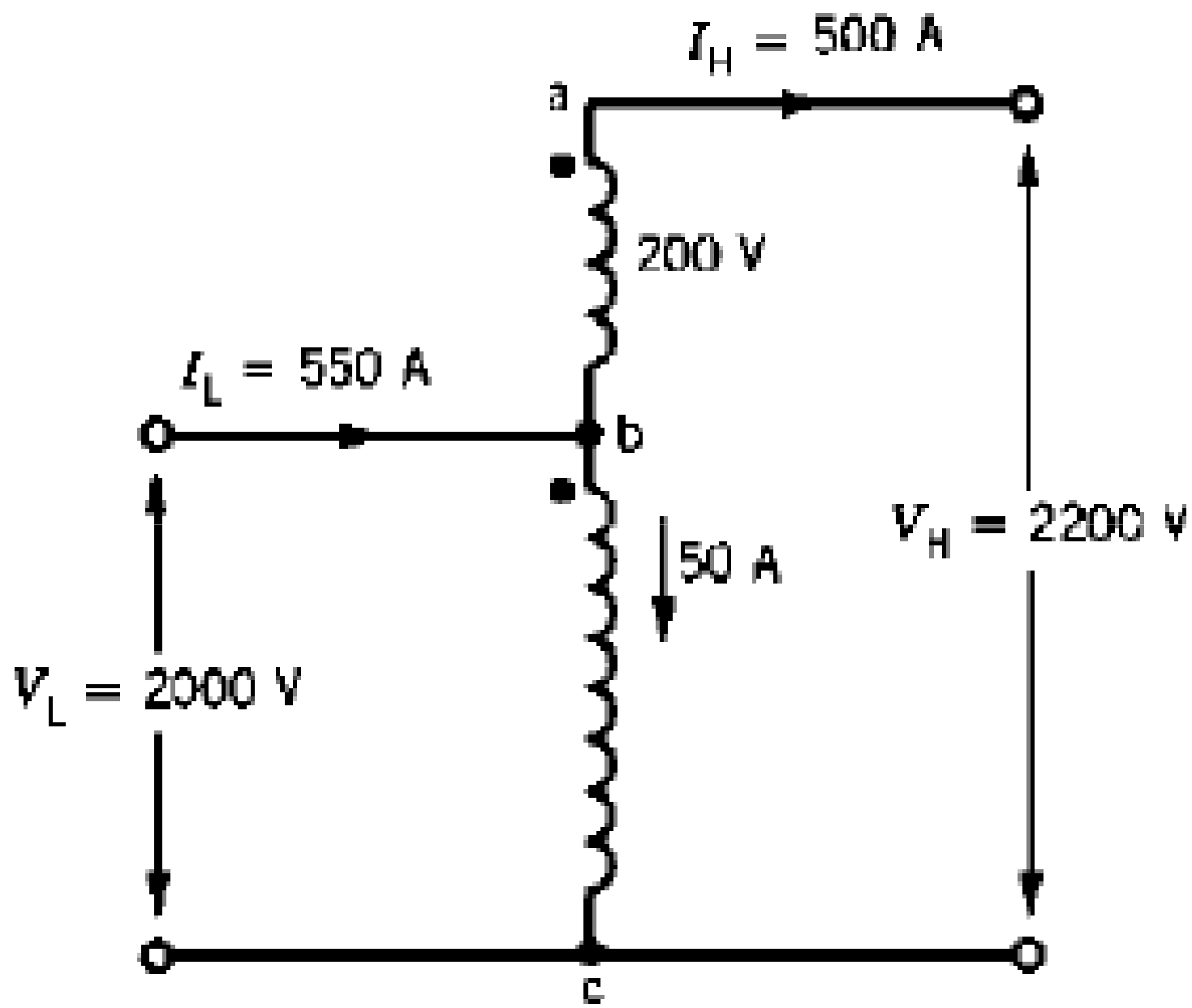
Application of Auto-transformer

- Boosting or bucking of a supply voltage by a small amount. (The smaller difference voltage between the output and input voltages the greater is the saving of winding material.)
- Starting of a.c. machines, e.g. induction motor, where the voltage is raised in two or more steps from a small value to the full supply voltage.
- Continuously variable a.c. supply voltages, normally connected between a low voltage supply in and a high voltage supply out.
- Production of very high voltages, e.g. 275kV and 400kV grid system

Example

A 1ϕ , 100 kVA, 2000/200 V two-winding transformer is connected as an autotransformer as shown in Fig. E2.6 such that more than 2200 V is obtained at the secondary. The portion **ab** is the 200 V winding, and the portion **bc** is the 2200 V winding. Compute the kVA rating as an autotransform

$$I = 500 \text{ A}$$



The current rating of the winding are:

$$I_{ab} = \frac{100,000}{200} = 500A$$

$$I_{bc} = \frac{100,000}{2,000} = 50A$$

Therefore, for full-load operation of the autotransformer, the terminal currents are:

$$I_H = 500A$$

$$I_L = 500 + 50 = 550A$$

Now,

Therefore,

$$\text{kVA}|_L = \frac{2000 \times 550}{1000} = 1100 \quad (\text{ans})$$

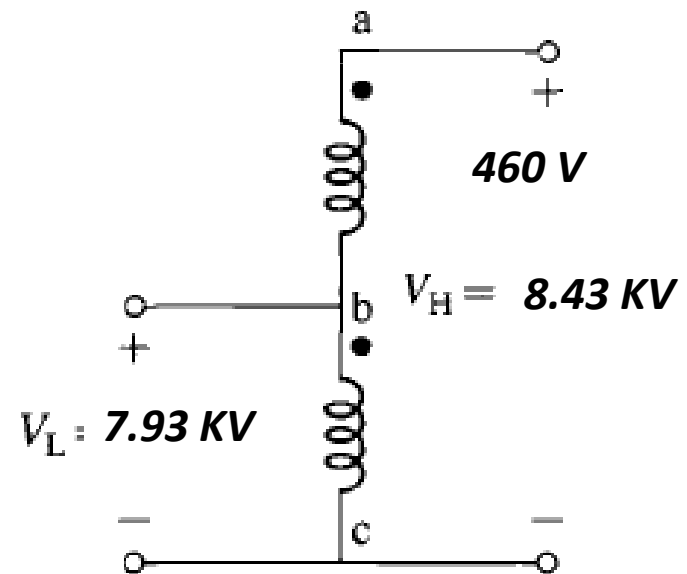
$$\text{kVA}|_H = \frac{2200 \times 500}{1000} = 1100 \quad (\text{ans})$$

Note: A 1ϕ , 100 kVA, two-winding transformer when connected as an autotransformer can deliver 1100 kVA.

Example

A 450-kVA, 460-V:7.97-kV transformer has an efficiency of 97.8 percent when supplying a rated load of unity power factor. If it is connected as a 7.97:8.43-kV autotransformer, calculate its rated terminal currents, rated kVA, and efficiency when supplying a unity-power-factor load.

- *The auto-transformer*
- *Connection is as shown.*
- *The rated two winding*
- *Transformer currents are:*



- $i_1 = \frac{450}{7.97} = 56.46 \text{ A}$
- $i_2 = \frac{450 \times 10^3}{460} = 978.26 \text{ A}$
- *Supply current $i_s = i_1 + i_2 = 1034.72 \text{ A}$*
- *Rating as an auto-transformer is:*
- $S_r = V_s i_s = 8246.71 \text{ KVA}$
- *Losses as a two winding transformer is:*
- $\text{losses} = S_2 \cdot pf \cdot \left(\frac{1}{\eta} - 1 \right) = 10.122 \text{ kw}$
- *Losses as an auto-transformer are the same as that of the two winding transformer, then*

- *Efficiency of auto-transformer is:*
- $\eta = \frac{8246.71}{8246.71+10.122} \times 100 = 99.87\%$
- **Note** *that the rating of the same single phase transformer when connected as an auto-transformer has been largely increased without over-loading of any of its windings. Only the insulation of the 460 v winding is to be increased to withstand the high voltage to ground impressed on it.*

Homework # 5

Transformers in three phase systems

- *Transformers in three phase systems can be either:*
- *1- Three single phase units of equal ratings, and equal rated voltages connected together to form three phase bank. In this case:*
- *The total rating of the bank is equal to three times the rating of each single phase unit.*
- *While the rated line voltages on each side of the bank depends upon type of connection used.*

- *If turns ratio of each single phase transformer is:*
- $a = \frac{N_1}{N_2}$
- *Then for the most common types of connections used in three phase systems, we got the following relations:*

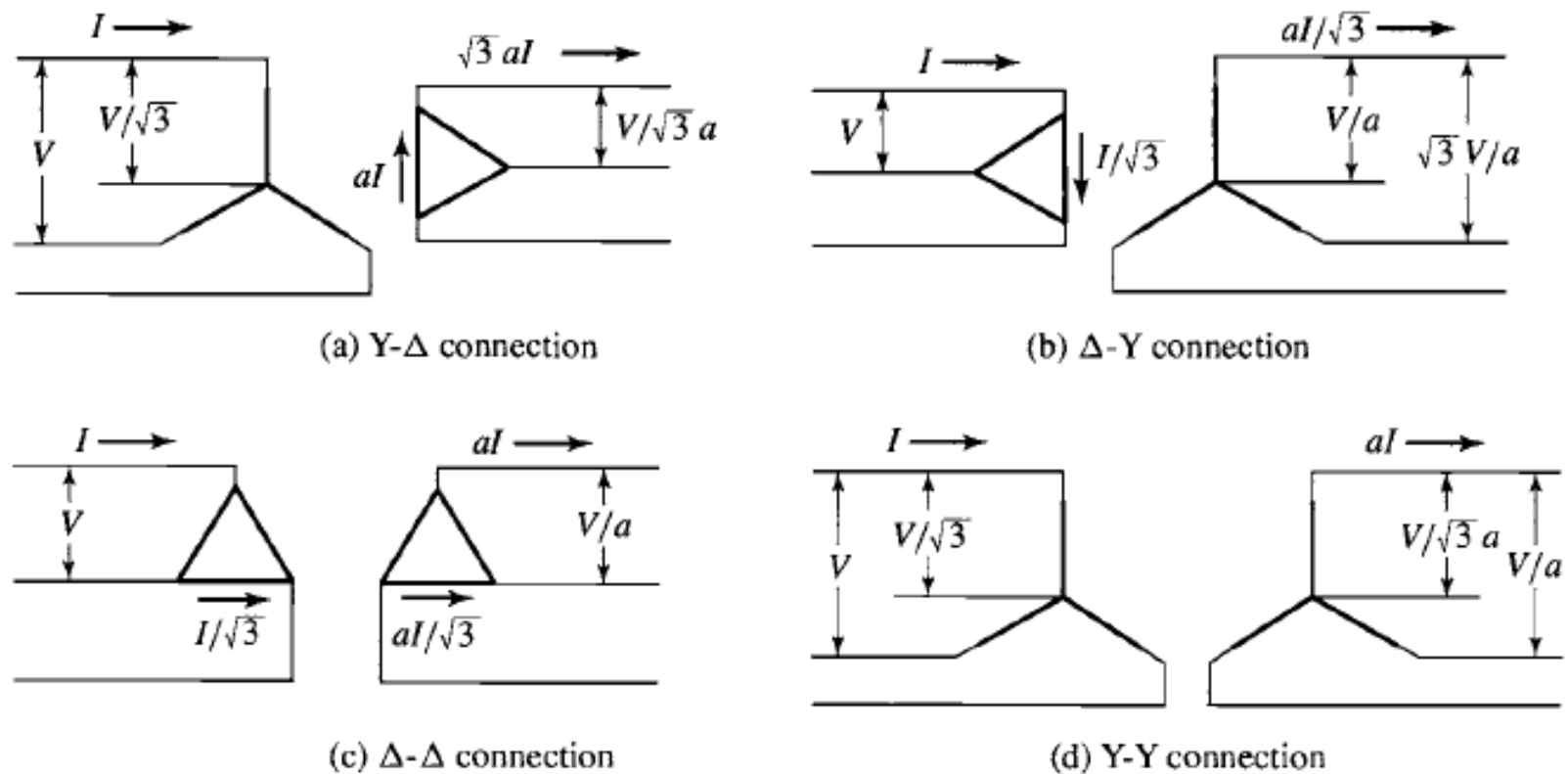


Figure 2.19 Common three-phase transformer connections; the transformer windings are indicated by the heavy lines.

Method of analysis

- *Assuming a balanced three phase system, operating at steady state conditions, then a per-phase analysis method can be used. That is by taking one phase of the y-connected side, and performing all analysis on it. The other phases will be similar to this phase except the phase difference that exists between these phases. This can be done as:*

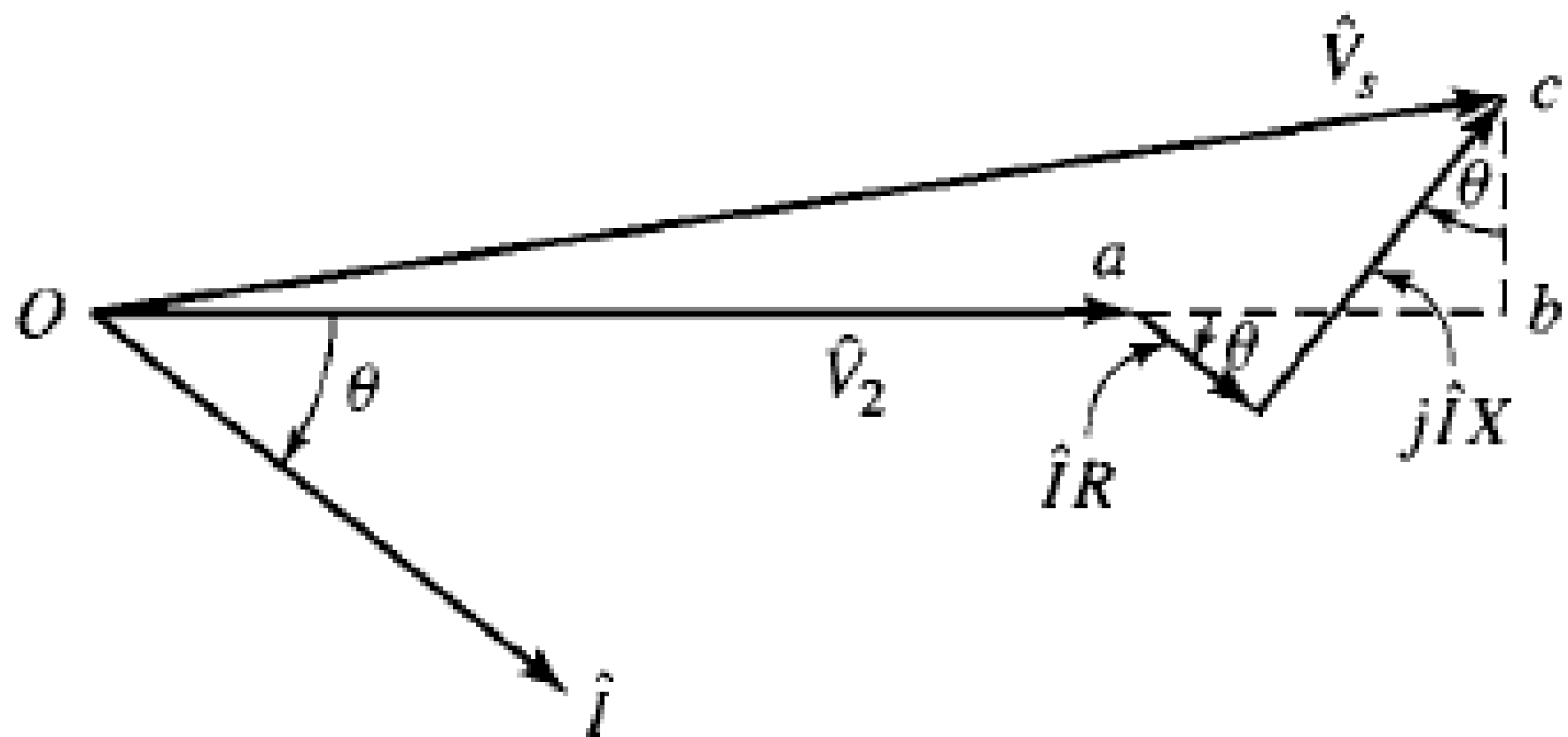
- 1- For y-y, y- Δ , and Δ -y connections refer all circuit parameters to the y-side.
- 2- For Δ - Δ connections, replace them by an equivalent y-y connection. Where:
- $Z_y = \frac{1}{3} Z_{\Delta}$

Example

- *Three single phase transformers each one is 50 KVA, 2400/240 V. The equivalent impedance of each transformer referred to its 2400 side is $1.42 + j1.82$ Ohms. The three transformers are connected $y-\Delta$ to form a three phase 150 KVA bank. The bank is connected to 4160 V supply through a feeder whose impedance is $0.15 + j 1.0$ Ohms. The bank supplies its rated load, at 0.8 lagging p.f to*

- *Load through a feeder whose impedance is $0.0005 + j 0.002$ Ohms. Determine the load voltage.*
- ***Solution:***
- *Refer all parameters to the y- connected side, such that per-phase analysis can be done:*
- $$Z_{f2} = \left(\frac{4160}{240} \right)^2 (0.0005 + j0.002) = 0.15 + j0.6 \Omega$$
- *Take the rated load per-phase voltage as reference, then:*

- $V'_2 = |V| \angle 0^\circ$, $I'_2 = 20.8 \angle -36.67^\circ \text{ A}$
- *The total impedance up to the supply is:*
- $Z_t = Z_{fh} + Z_{th} + Z_{f2h} = 1.72 + j 3.42 \text{ Ohms}$
- *The magnitude of the supply phase voltage is 2400 V.*
- *Then using the phase diagram, we get:*



From the phasor diagram

$$Ob = \sqrt{V_s^2 - (bc)^2} \quad \text{and} \quad V_2 = Ob - ab$$

Note that

$$bc = IX \cos \theta - IR \sin \theta \quad ab = IR \cos \theta + IX \sin \theta$$

where R and X are the combined resistance and reactance, respectively. Thus

$$bc = 20.8(3.42)(0.80) - 20.8(1.72)(0.60) = 35.5 \text{ V}$$

$$ab = 20.8(1.72)(0.80) + 20.8(3.42)(0.60) = 71.4 \text{ V}$$

Substitution of numerical values shows that $V_2 = 2329 \text{ V}$, referred to the high-voltage side. The actual voltage at the secondary terminals is $2329/10$, or

$$V_2 = 233 \text{ V}$$

Example

The three transformers of Example 2.8 are reconnected Δ - Δ and supplied with power through a 2400-V (line-to-line) three-phase feeder whose reactance is $0.80 \text{ } \Omega/\text{phase}$. At its sending end, the feeder is connected to the secondary terminals of a three-phase Y- Δ -connected transformer whose rating is 500 kVA, 24 kV:2400 V (line-to-line). The equivalent series impedance of the sending-end transformer is $0.17 + j0.92 \text{ } \Omega/\text{phase}$ referred to the 2400-V side. The voltage applied to the primary terminals of the sending-end transformer is 24.0 kV line-to-line.

A three-phase short circuit occurs at the 240-V terminals of the receiving-end transformers. Compute the steady-state short-circuit current in the 2400-V feeder phase wires, in the primary and secondary windings of the receiving-end transformers, and at the 240-V terminals.

The computations will be made on an equivalent line-to-neutral basis with all quantities referred to the 2400-V feeder. The source voltage then is

$$\frac{2400}{\sqrt{3}} = 1385 \text{ V line-to-neutral}$$

- *Change the delta/delta transformer to equivalent star/star as:*

$$Z_{eq} = R_{eq} + jX_{eq} = \frac{1.42 + j1.82}{3} = 0.47 + j0.61 \Omega/\text{phase}$$

The total series impedance to the short circuit is then the sum of this impedance,

$$Z_{tot} = (0.47 + j0.61) + (0.17 + j0.92) + j0.80 = 0.64 + j2.33 \Omega/\text{phase}$$

$$|Z_{tot}| = 2.42 \Omega/\text{phase}$$

The magnitude of the phase current in the 2400-V feeder can now simply be calculated as the line-neutral voltage divided by the series impedance

- *The short circuit current is:*

$$\text{Current in 2400-V feeder} = \frac{1385}{2.42} = 572 \text{ A}$$

the winding current in the 2400-V winding of the receiving-end transformer is equal to the phase current divided by $\sqrt{3}$ or

$$\text{Current in 2400-V windings} = \frac{572}{\sqrt{3}} = 330 \text{ A}$$

while the current in the 240-V windings is 10 times this value

$$\text{Current in 240-V windings} = 10 \times 330 = 3300 \text{ A}$$

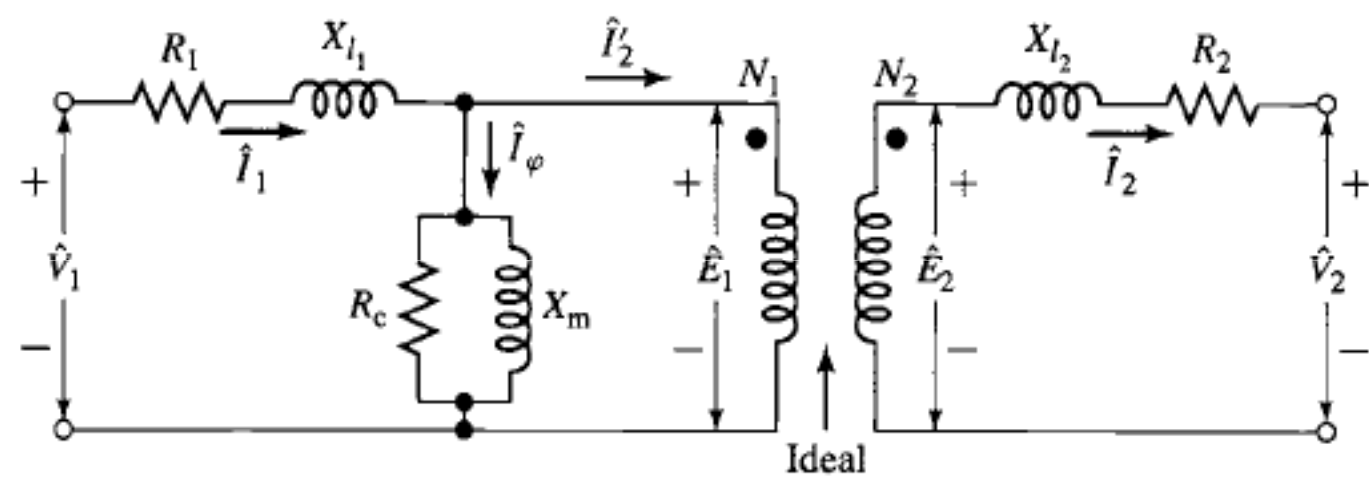
- *At the load side:*

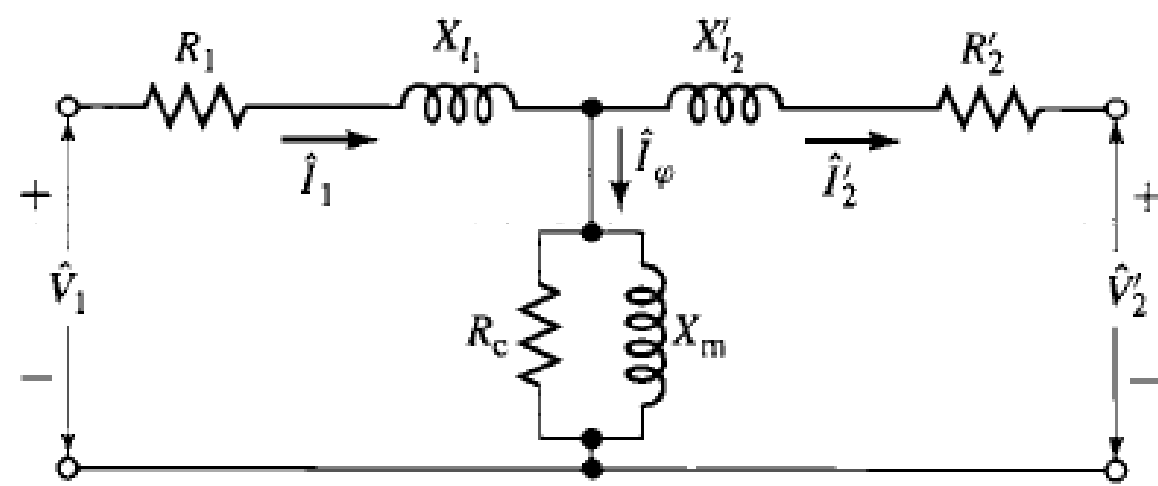
the phase current at the 240-V terminals into
the short circuit is given by

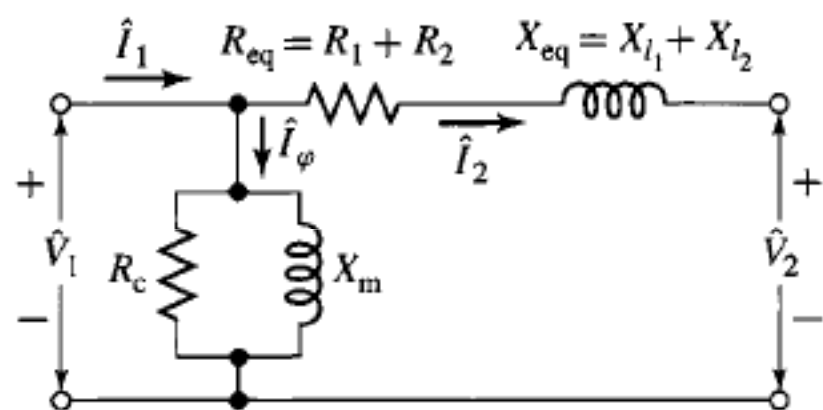
$$\text{Current at the 240-V terminals} = 3300\sqrt{3} = 5720 \text{ A}$$

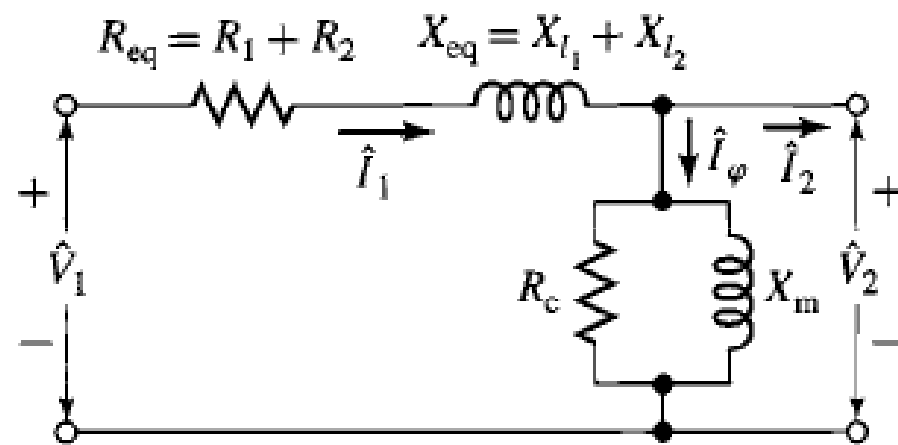
Note of course that this same result could have been computed simply by recognizing that the turns ratio of the Δ - Δ transformer bank is equal to 10:1 and hence, under balanced-three-phase conditions, the phase current on the low voltage side will be 10 times that on the high-voltage side.

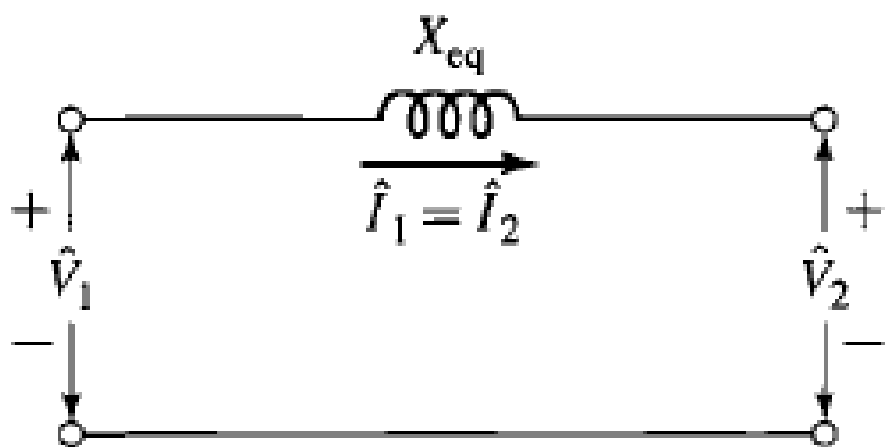
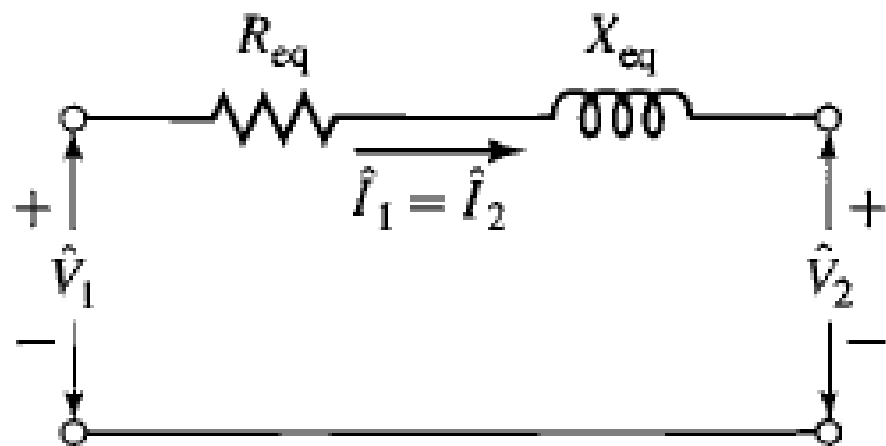
Homework # 5











Solution

